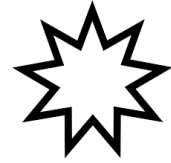


Problem of the Week

Problem B and Solution

Nein!

**Problem**

In the table, the first column contains a number n , where n goes from 2 to 10.

Complete the table as follows:

- in the next two columns, write the tens digit and the ones digit of the product p where $p = 9 \times n$;
- in the fourth column, write the difference, d , between the number n and the tens digit of p . That is,
 $d = n - \text{tens digit of } p$; and
- in the last column, write the sum, S , of the tens and ones digits of the product p .

n	$p = 9 \times n$		difference d	sum S
	tens	ones		
2	1	8	$2 - 1 = 1$	$1 + 8 = 9$
3	2	7	$3 - 2 = 1$	$2 + 7 = 9$
4	3	6	$4 - 3 = 1$	$3 + 6 = 9$
5	4	5	$5 - 4 = 1$	$4 + 5 = 9$
6	5	4	$6 - 5 = 1$	$5 + 4 = 9$
7	6	3	$7 - 6 = 1$	$6 + 3 = 9$
8	7	2	$8 - 7 = 1$	$7 + 2 = 9$
9	8	1	$9 - 8 = 1$	$8 + 1 = 9$
10	9	0	$10 - 9 = 1$	$9 + 0 = 9$

Answer the following questions by making observations from the completed table.

- What is a good way to find the tens digit of the product p ?
- What is a good way to determine the ones digit of the product p ?
- What other interesting patterns do you see in the table?

Solution

- The completed table above reveals that the tens digit of a multiple of 9 is one less than the number by which you are multiplying 9. For example, $9 \times 3 = 27$, has the tens digit of $3 - 1 = 2$.
- To find the ones digit of a multiple of 9, once you have found the tens digit, just subtract that tens digit from 9. For example, $9 \times 3 = 27$ has a tens digit of 2 and thus the ones digit is $9 - 2 = 7$.
- All the differences $d = n - \text{tens digit}$ are equal to 1. Also, the sum tens digit + ones digit is equal to 9 in all cases.

To think about: What happens if you look at further products of 9? Do any of these patterns persist? Which ones?

