Problem of the Week

Problem B and Solution

Nein!

Problem

In the table, the first column contains a number n, where n goes from 2 to 10.

Complete the table as follows:

- in the next two columns, write the tens digit and the ones digit of the product p where $p = 9 \times n$;
- in the fourth column, write the difference, d, between the number n and the tens digit of p. That is,
  $d = n -$ tens digit of p; and
- in the last column, write the sum, S, of the tens and ones digits of the product p.

	$p = 9 \times n$		difference	sum
n	**tens**	**ones**	d	S
2	1	8	$2-1=1$	$1+8=9$
3	2	7	$3-2=1$	$2+7=9$
4	3	6	$4-3=1$	$3+6=9$
5	4	5	$5-4=1$	$4+5=9$
6	5	4	$6-5=1$	$5+4=9$
7	6	3	$7-6=1$	$6+3=9$
8	7	2	$8-7=1$	$7+2=9$
9	8	1	$9-8=1$	$8+1=9$
10	9	0	$10-9=1$	$9+0=9$

Answer the following questions by making observations from the completed table.

a) What is a good way to find the tens digit of the product p?

b) What is a good way to determine the ones digit of the product p?

c) What other interesting patterns do you see in the table?

Solution

a) The completed table above reveals that the tens digit of a multiple of 9 is one less than the number by which you are multiplying 9. For example, $9 \times 3 = 27$, has the tens digit of $3 - 1 = 2$.

b) To find the ones digit of a multiple of 9, once you have found the tens digit, just subtract that tens digit from 9. For example, $9 \times 3 = 27$ has a tens digit of 2 and thus the ones digit is $9 - 2 = 7$.

c) All the differences $d = n -$ tens digit are equal to 1. Also, the sum tens digit + ones digit is equal to 9 in all cases.

To think about: What happens if you look at further products of 9? Do any of these patterns persist? Which ones?