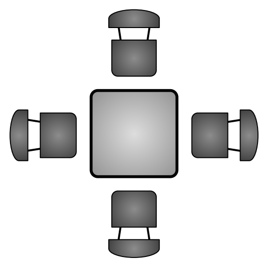
Problem of the Week

Problem A and Solution

Dining Dilemma

Problem

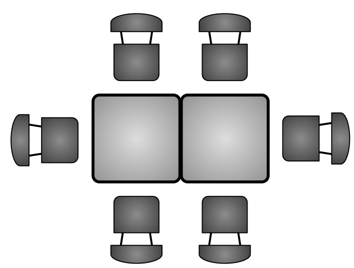
We want to start a new restaurant. We have square tables that allow one chair on each side. Therefore, we can arrange four chairs around each table.



A) If the restaurant has 32 tables, how many chairs do we need to buy?

B) As we set up the restaurant, we put out one table at a time with its full set of chairs surrounding it. If we have put out 36 chairs, how many tables have been set up so far?

C) When we have banquets we sometimes need to push the tables together. This changes the amount of chairs we can put around the table grouping, as shown in the following picture.



How many chairs are required if we set up the 32 tables in pairs?

D) How would the answer to part (C) change if we group 8 tables end to end, and still use all 32 tables?

Solution

A) We can use multiplication to calculate the number of chairs: $32 \times 4 = 128$.

We might also notice that multiplying by 4 is the same as doubling a number and then doubling the answer. For example, we can find the answer to 32×4 by calculating $32 \times 2 = 64$ and then calculating $64 \times 2 = 128$.

B) We can use skip counting to find out how many tables are out.
We skip count by 4 to count the chairs: *4, 8, 12, 16, 20, 24, 28, 32, 36*. This means there are 9 tables set up so far. We could have also used division to calculate this answer: $36 \div 4 = 9$.

C) We can make a table showing the relationship between the number of tables and the number of chairs using this configuration. Each cluster of 2 tables is surrounded by 6 chairs. So we add 2 to the number of tables from one row to the next, and we add 6 to the number of chairs from one row to the next.

Number of Tables	Number of Chairs
2	6
4	12
6	18
8	24
10	30
12	36
14	42
16	48
18	54
20	60
22	66
24	72
26	78
28	84
30	90
32	96

Another way to calculate this result is as follows:

Since there are 32 tables in total, there are $32 \div 2 = 16$ pairs of tables. In this configuration, there 6 chairs around each pair of tables. Therefore, there are a total of $16 \times 6 = 96$ chairs required for this set up.

D) With 8 tables arranged end to end, there will be one chair on each end, and eight chairs on each side. This is a total of $1 + 1 + 8 + 8 = 18$ chairs around the tables.

We can make a new table showing the relationship between the number of tables and the number of chairs using this configuration. In this case we add 8 to the number of tables from one row to the next, and we add 18 to the number of chairs from one row to the next.

Number of Tables	Number of Chairs
8	18
16	36
24	54
32	72

Another way to calculate this result is as follows:

Since there are 32 tables in total, there are $32 \div 8 = 4$ sets of 8 tables. In this configuration, there 18 chairs around each pair of tables. Therefore, there are a total of $18 \times 4 = 72$ chairs required for this set up.

Teacher's Notes

Part of the solutions for parts C and D can be described as a *table of values* (no pun intended). A table of values can be used to list specific values that are described by a function. We write functions in a variety of ways. For example, we can write a function describing the values in part D in the form of an equation that uses a variable c to represent the number of chairs and a variable t that represents the number of tables. This equation would be:

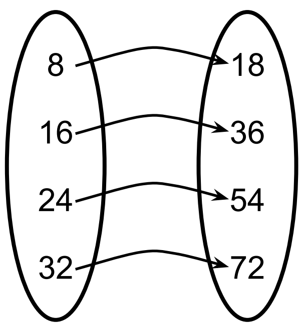
$$c = \frac{t}{8} \times 18$$

Another way of describing this function uses x to represent the number of tables and $f(x)$ to represent the number of chairs. The function that uses this notation would be:

$$f(x) = \frac{x}{8} \times 18$$

Normally when we use these formats to describe functions, we expect that the variables in the equations represent many possible numbers - sometimes infinitely many numbers. However, for this problem there is a *finite* set of possible values for the number of tables we arrange in the restaurant. In particular, we only consider groupings of 8, 16, 24, or 32 tables. So numbers like 1, 2, 3, ..., 7, 9, 10 and so on are not actual values that would be used for t or x, in the functions we have written.

Another way we describe functions is called a *mapping*. This is a visual representation of the relationship between two sets of numbers. For example, we use a mapping like this to describe the values in part D of this problem:



In a mapping, an oval represents a *set* of values. In this example, the oval on the left represents the set of values representing the number of tables we may group together in this problem. The oval on the right represents the set of values that are the related number of chairs. Values in the set on the left are connected, using an arrow, to a particular value in the set on the right. Altogether, this picture represents a function that describes the relationship between 8, 16, 24, or 32 tables and the number of chairs required in each case. A mapping is a nice way to represent a function with a finite set of values.