Problem of the Week
Problem A and Solution
Ups and Downs

Problem

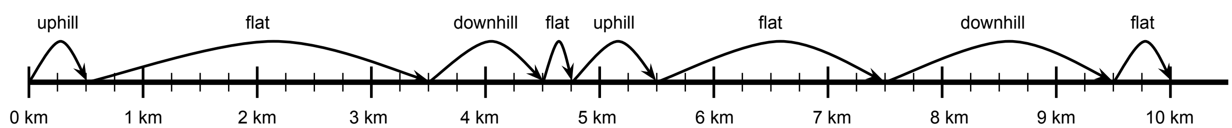
Graham rides his bike to school. He takes a different route home. On the way to school he rides up a hill for 500 metres, then he rides on a flat section for 3 kilometres, and then he rides downhill for 1 kilometre. On his way home he rides on a flat section for 250 metres, then he rides uphill for 750 metres, followed by another flat section for 2 kilometres, then downhill for 2 kilometres, and finally on a flat section for 500 metres.

A) How far does Graham ride to and from school every day?

B) What fraction of the total distance travelled in one day is Graham riding downhill?

Solution

A) We can use a number line to calculate the total distance Graham travelled. Each interval on this line represents 250 m.



From the number line we can see that Graham travels a total of 10 km.

We could also add the individual distances together to determine the total. It is probably easier to do this if all of the distances were measured with the same unit. So, converting all of the distances to metres, we get:

$$500 + 3000 + 1000 + 250 + 750 + 2000 + 2000 + 2000 + 500 = 10000$$

and 10000 m is equal to 10 km.

B) Graham travels 1 km downhill on the way to school and 2 km downhill on the way home. This is a total of $1 + 2 = 3$ km.

Since he travels a total of 10 km in one day, the downhill portion of the distance he travels is: $\frac{3}{10}$.

Teacher's Notes

In this solution we use a number line to accumulate the distance Graham travels. To make the number line a useful tool, we must make a good choice for the size of its intervals. A simple number line might have each interval be equal to one unit, however that scale does not work in this case. If we had each interval represent 1 km, then it would be hard to accurately accumulate fractional values like 250 m or 500 m. If we had each interval represent 1 m, then the number line would either be extremely long or the spaces between intervals would be extremely small. Even choosing 100 m as the distance between intervals would not be the best choice, since some of the accumulated values would land between the tick marks on the number line.

One way to pick the interval size is to consider the *greatest common divisor* or *GCD* of the numbers that we are accumulating. The *GCD* of a set of numbers is the largest integer that divides evenly into each of the numbers in the set. In this case, we have the numbers 500, 3000, 1000, 250, 750, and 2000. For all of these numbers, we get an integer result when we divide them by 250. By choosing an interval size that is a divisor of each of the numbers we are accumulating, we guarantee that accumulated values will always land on one of the tick marks of our number line. Choosing the interval size to be the greatest common divisor, means we have the fewest intervals necessary to guarantee accumulated values will always end up landing on a tick mark.

In this case, the *GCD* also happens to be the smallest number in our set. However, that is not always the case. For example, the *GCD* of the numbers 400, 2000, and 900 is 100. The *GCD* will always be less than or equal to the smallest number in the set and greater than or equal to 1.