Problem of the Week
Problem A and Solution
Ship Flip

Problem
A transformation is the result of moving a shape according to a rule. There are three basic transformations: translate (slide), rotate (turn) and reflect (flip).

A sailing ship that is shown in the diagram on the right has one side that is simply an outline of the image. The other side of the ship, which is not shown, is filled in (black).

Identify which of the following images are a result of one or more transformations of our original image? Explain your reasoning.

A. B. C. D.

Solution
Note that a flip of the original image would cause the black side to show.

- Image A is possible by rotating the original image by a half turn ($180^\circ$).
- It is not possible to transform the original image into image B.
- Image C is possible by flipping the original image vertically, and rotating it by quarter turn ($90^\circ$) counterclockwise.
- It is not possible to transform the original image into image D.

So images A and C result from transforming our original image.
Teacher’s Notes

There is more than one way to transform the original image to the images A and C. Generally, we should expect that there are multiple paths from a starting image to a transformed image. For example, it is possible to generate image C by doing a clockwise quarter turn rotation first and then a vertical flip. We could also get to that final image by doing a counterclockwise quarter turn followed by a horizontal flip.

We always have two choices when rotating images to get the same result. A rotation of $x^\circ$ clockwise is equal to $(360 - x^\circ)$ counterclockwise. So image C can be generated by doing a vertical flip followed by a $270^\circ$ turn clockwise. Also, a rotation of $180^\circ$ is equivalent to a horizontal flip followed by a vertical flip or a vertical flip followed by a horizontal flip. So we can see that there are multiple ways to transform the original image to image A.