Problem of the Week
Problem E and Solution
The Truth of the Matter

Problem
Four people, Andy, Barb, Carl and Dana, each said two statements such that:
- one person lied in both statements;
- one person told the truth in both statements; and
- two people told the truth in one statement and a lie in the other statement.

Andy said, “Barb lied once” and “Dana lied twice.” Barb said, “I never lie” and “Andy never lied.” Carl said, “Dana lied twice” and “Barb never lied.” Dana said, “Andy lied twice” and “I never lie.”

Who lied twice? Who never lied? Who lied exactly once?

Solution

1. Who lied twice?
   a) Assume that Andy lied twice. If so, “Barb lied once” is a lie. Therefore, “Barb never lied” or “Barb lied twice.” But “Barb never lied” cannot be true because Barb says, “Andy never lied.” This contradicts our assumption that Andy lied twice. “Barb lied twice” cannot be true since that means Barb and Andy both lied twice and this contradicts the fact that only one person lied twice. Therefore, our assumption that Andy lied twice is false.
   
   b) Assume that Barb lied twice. If so, “Andy never lied” is a lie. Then, Andy lied twice or Andy lied once. Andy cannot have lied twice since both he and Barb would have lied twice and this contradicts the fact that only one person lied twice. But “Andy lied once” is also false since “Barb lied once” is a lie (we assumed she lied twice) and “Dana lied twice” is a lie because it contradicts the assumption that Barb lied twice (and only one person can lie twice). Therefore, our assumption that Barb lied twice is false.
   
   c) Assume that Carl lied twice. If so, “Barb never lied” is a lie and “Dana lied twice” is a lie. Since “Barb never lied” is a lie, then she lied twice or she lied once. But if Barb lied twice our assumption that Carl lied twice cannot be true since only one person lied twice.

   If Barb lied once, then “I never lie” must be the lie and “Andy never lied” must be true. But if Andy never lied, then “Dana lied twice” must be true and this contradicts the fact that only one person can lie twice. Therefore, our assumption that Carl lied twice is false.

   We are told that one person lied twice and none of Andy, Barb or Carl lied twice. Therefore, by elimination, Dana is the one who lied twice.
2. Who never lied?

   a) Assume that Barb never lied. Then her statement that “Andy never lied” must be true. There are then two people who never lied. This contradicts the fact that only one person never lied. Therefore, our assumption that Barb never lied is false.

   b) Assume that Carl never lied. Then his statement that “Barb never lied” must be true. There are then two people who never lied. This contradicts the fact that only one person never lied. Therefore, our assumption that Carl never lied is false.

   Dana lied twice. Barb and Carl lied. Therefore, by elimination, Andy is the one who never lied. It then follows that Barb and Carl each make one true statement and tell one lie.

We can now check our results.

Andy never lied. Then his statements are both true. Barb lied once is true and Dana lied twice is true.

Barb lied once. Then one of her statements is true and the other is a lie. Her statement that she never lies is a lie and her statement that Andy never lies is true.

Carl lied once. Then one of his statements is true and the other is a lie. His statement that Dana lied twice is true and his statement that Barb never lied is a lie.

Dana lied twice. Then both of her statements are lies. Andy lied twice is a lie. And her statement the she never lies is a lie.

TRUE OR FALSE

QR Code