# Problem of the Week
## Problem E and Solution
## Faster Adder Required

### Problem

Determine the sum of all the three-digit numbers that can be made by choosing three different numbers from the list $\{1, 2, 3, 4, 5, 6, 7\}$.

## Solution

We need to first determine how many possible three-digit numbers can be formed using three different digits from the list $\{1, 2, 3, 4, 5, 6, 7\}$. There are 7 choices for the first digit. For each of these choices, there are 6 choices for the second digit giving a total of $7 \times 6 = 42$ choices for the first two digits. For each of these 42 choices for the first two digits, there are 5 choices for the third digit giving a total of $42 \times 5 = 210$ different three-digit numbers.

Each of the numbers 1 to 7 has an equal chance of appearing in each of the hundreds, tens and ones positions. Therefore, each digit appears $210 \div 7 = 30$ times in each place value position.

The sum of the digits in the units position is

$$\begin{aligned} & 30(1) + 30(2) + 30(3) + 30(4) + 30(5) + 30(6) + 30(7) \\ = \; & 30(1 + 2 + 3 + 4 + 5 + 6 + 7) \\ = \; & 30(28) \\ = \; & 840 \end{aligned}$$

The units digit of the sum is 0 and 84 is carried to the tens digit column.

The same digits appear in the tens digit column of the sum and again 30 times each. So the sum of the tens digit column is 924 which is the sum of the digits in the column plus 84 carried from the units digit column. The tens digit of the sum is 4 and 92 is carried to the hundreds digit column.

The same digits appear in the hundreds digit column of the sum and again 30 times each. So the sum of the hundreds digit column is 932 which is the sum of the digits in the column plus 92 carried from the tens digit column. The required sum is therefore 93 240.

The same sum would be obtained by adding 111, 222, 333, 444, 555, 666 and 777, and multiplying the sum by 30.

$$\begin{aligned} & 30(111 + 222 + 333 + 444 + 555 + 666 + 777) \\ = \; & 30 \times 111 \times (1 + 2 + 3 + 4 + 5 + 6 + 7) \\ = \; & 3330(28) \\ = \; & 93\,240 \end{aligned}$$

It is left to the solver to reason this out.