

## Problem of the Week

### Problem E

#### Clipping Along

Gwen and Chris are playing a game. They begin with a pile of paperclips, and use the following rules.

1. The two players alternate turns.
2. On any turn, a player can remove 1, 2, 3, 4, or 5 paperclips from the pile.
3. The same number of paperclips cannot be removed on two different turns during the entire game.
4. The last person who is able to play wins, regardless of whether there are any paperclips remaining in the pile after their turn.

For example, if a game begins with 9 paperclips, then the following moves could occur. Gwen removes 2 paperclips, leaving 7 in the pile. Chris removes 5 paperclips, leaving 2 in the pile. Gwen removes 1 paperclip, leaving 1 in the pile. Gwen is now the winner, since Chris cannot remove 1 paperclip. (Gwen already removed 1 paperclip on one of her turns, and the third rule states that a person cannot remove the same number that has already been removed.)

Suppose the game starts with 8 paperclips and Gwen goes first. Find all initial moves that Gwen can make to guarantee that she will win. Justify your answers.

