Problem of the Week
Problem E
Useful Facts Indeed!

A *prime* number is a positive integer greater than 1 that has exactly two positive integer factors, 1 and the number itself. A *composite* number is a positive integer greater than 1 that has more than two positive integer factors. The number 1 is neither prime nor composite.

For some number $21609d$, with units digit d, $2^{21609d} - 1$ is a very large prime number.

In fact, the number contains 65 050 digits. The number begins 746 093 103 064 661 343 $\cdots$ and ends with the units digit 7.

Determine the value of d, the units digit of $21609d$.

Here are some useful facts which may be helpful in solving this problem:

1. if n is divisible by 3, then $2^n - 1$ is divisible by 7; and

2. if n is divisible by 5, then $2^n - 1$ is divisible by 31.

One use for very large prime numbers is in the area of *cryptography*, the study of coding and decoding information so that it can be securely transmitted. This area of study is very important because of its application to areas like online banking, email, and general internet security, to list just a few.

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