Problem of the Week

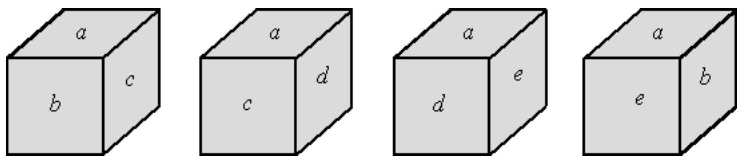
Problem E and Solution

A Different Point of View

Problem

Each of the numbers 1, 2, 3, 4, 5, 6 occurs, one to a face, on the faces of a cube. Three people, Bel, Cal and Dan, are seated around a rectangular table. Bel is seated on one side of the table. Cal is seated on the side of the table which is adjacent to Bel and to her right. Dan is seated on the side of the table which is adjacent to Cal and to his right. There is an empty seat along the side which is adjacent to both Bel and Dan. The cube is placed on the table so that from their different seat locations, each one can see the top face and two adjacent side faces. When Bel adds the three numbers that she can see, her total is 9. When Cal adds the three numbers that he can see, his total is 14. When Dan adds the three numbers that he can see, his total is 15. Determine the number on the bottom face of the cube.

Solution



Let a represent the number on the top face of the cube.

Let a, b and c represent the three numbers that Bel sees.

Let a, c and d represent the three numbers that Cal sees.

Let a, d and e represent the three numbers that Dan sees.

If there were a person in the fourth seat, that person would see a, e and b.

From the given information, we are now able to form three equations:

$$\begin{aligned} a+b+c &= 9 \quad (1) \\ a+c+d &= 14 \quad (2) \\ a+d+e &= 15 \quad (3) \end{aligned}$$

Comparing equation (1) and equation (2), b has been replaced by d and the sum has increased by 5. Therefore, b and d differ by 5 and $b < d$. The only numbers from the set 1, 2, 3, 4, 5, 6 that differ by 5 are 1 and 6. Therefore, $b = 1$ and $d = 6$.

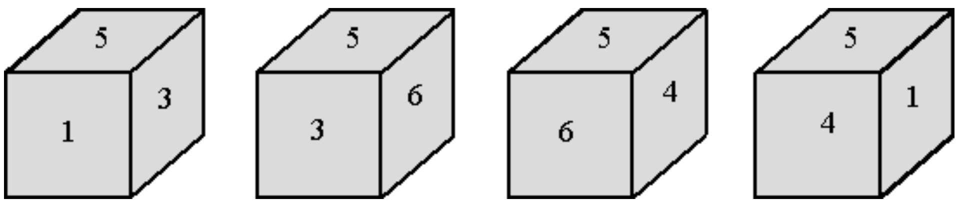
Comparing equation (2) and equation (3), c has been replaced by e and the sum has increased by 1. Therefore, c and e differ by 1 and $c < e$. Since $b = 1$ and $d = 6$, there are only three possible combinations for c and e, namely $c = 2$ and $e = 3$, or $c = 3$ and $e = 4$, or $c = 4$ and $e = 5$.

We will check each of these possibilities. First, if $c = 2$, $e = 3$, $b = 1$ and $d = 6$, we can substitute the appropriate values in (1) giving $a + 1 + 2 = 9$ or $a = 6$. This is not possible since d would also equal 6. We can rule this case out.

Next, if $c = 3$, $e = 4$, $b = 1$ and $d = 6$, we can substitute the appropriate values in (1) giving $a + 1 + 3 = 9$ or $a = 5$. This is a possible solution. In (2), $a + c + d = 5 + 3 + 6 = 14$ as required. And in (3), $a + d + e = 5 + 6 + 4 = 15$ as required. The only number not used is 2 so the number on the bottom face is 2. But is this the only solution?

Finally, if $c = 4$, $e = 5$, $b = 1$ and $d = 6$, we can substitute the appropriate values in (1) giving $a + 1 + 4 = 9$ or $a = 4$. This is not possible since c would also equal 4. We can rule this case out.

Since we have examined all possible cases, the only possible number on the bottom (unseen) face is 2.



Notes:

It is also possible to play with the numbers to solve this problem. The method presented above could be used in a similar way with any list of six different numbers. "Playing" with the numbers might not be as easy.

Instead of "arguing" the difference between equations to obtain the relationship between b and d, and c and e, we could have used elimination.

$$\begin{aligned} a + b + c &= 9 \quad (1) \\ a + c + d &= 14 \quad (2) \\ a + d + e &= 15 \quad (3) \end{aligned}$$

For example, equation (1) subtract equation (2) gives $b - d = -5$ which can be written $d - b = 5$. This is the same as saying the difference between b and d is 5.

Similarly, equation (2) subtract equation (3) gives $c - e = -1$ which can be written $e - c = 1$. This is the same as saying that the difference between c and e is 1.