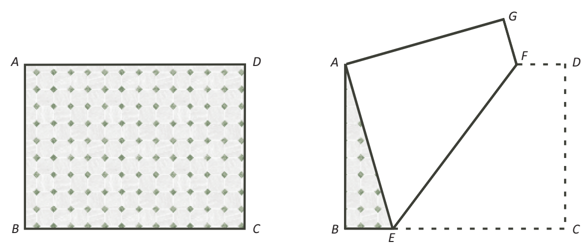
Problem of the Week
Problem E and Solution
Paper Folding 101

Problem

The problem today involves simple paper folding. In fact, only one fold is required.

A rectangular piece of paper is 30 cm wide and 40 cm long. The paper has a pattern on one side and is plain on the other. The paper is folded so that the two diagonally opposite corners, A and C, coincide. (This is illustrated on the diagram to the right.)



Determine the length of the crease, FE, created by the fold.

Solution

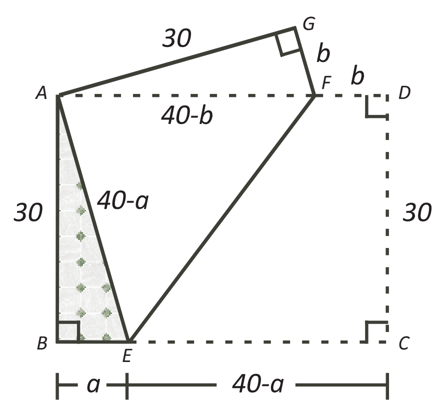
After the fold, C coincides with A and D folds to G. The angle at G is the same as the angle at D. Since $ABCD$ is a rectangle, $\angle ADC = 90^\circ$ and it follows that $\angle AGF = 90^\circ$.

Let a represent the length of BE and b represent the length of FD.
Then $EC = CB - BE = 40 - a$ and $AF = AD - FD = 40 - b$.

The distance from the top of the crease at F to D is the same length as the distance from F to G. It follows that $FG = FD = b$.

The distance from the bottom of the crease at E to C is the same length as the distance from E to A. It follows that $AE = EC = 40 - a$.

All of the information is recorded on the following diagram.

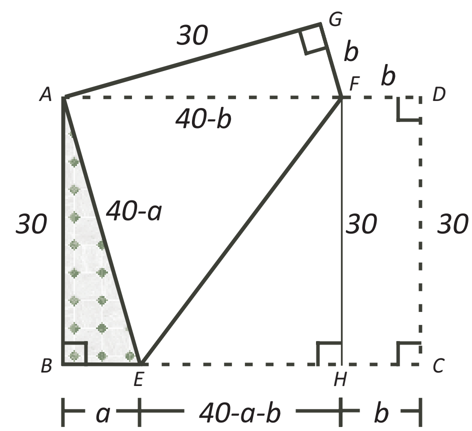


Since $\triangle ABE$ and $\triangle AGF$ are both right angled, we can use the Pythagorean Theorem to find a and b.

$$\begin{aligned} BE^2 + AB^2 &= AE^2 \\ a^2 + 30^2 &= (40-a)^2 \\ a^2 + 900 &= 1600 - 80a + a^2 \\ 80a &= 700 \\ a &= \frac{35}{4} \end{aligned} \qquad \text{and} \qquad \begin{aligned} FG^2 + AG^2 &= AF^2 \\ b^2 + 30^2 &= (40-b)^2 \\ b^2 + 900 &= 1600 - 80b + b^2 \\ 80b &= 700 \\ b &= \frac{35}{4} \end{aligned}$$

$$\therefore a = b = \frac{35}{4}$$

We still need to find the length of the crease.



From F drop a perpendicular to BC intersecting at H. $FHCD$ is a rectangle. It follows that $FH = DC = 30$ and $HC = FD = b$.

Also, $EH = BC - BE - HC = 40 - a - b = 40 - \frac{35}{4} - \frac{35}{4} = \frac{90}{4} = \frac{45}{2}$.

Using the Pythagorean Theorem in $\triangle EFH$,

$$\begin{aligned} EF^2 &= FH^2 + EH^2 \\ &= 30^2 + \left(\frac{45}{2}\right)^2 \\ &= 900 + \frac{2025}{4} \\ &= \frac{5625}{4} \\ EF &= \frac{75}{2} \quad (EF > 0) \end{aligned}$$

The length of the crease is $\frac{75}{2}$ cm (37.5 cm).