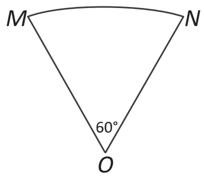


Problem of the Week

Problem E and Solution

Circle This

Problem

MON is a sector of a circle with radius ON which is 6 cm long. If $\angle MON = 60°$, determine the radius of the circle which passes through the points M, N, and O.

Solution

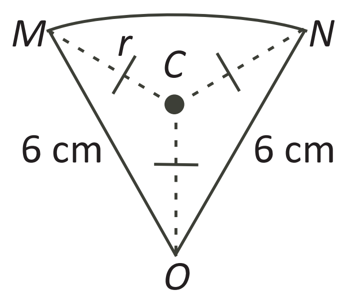
Let C be the centre of the circle that passes through M, N, and O. Then CM, CN, and CO are radii. Therefore, $CM = CN = CO = r$.

In $\triangle CMO$ and $\triangle CNO$, $CM = CN$, CO is common and $OM = ON$. Therefore, $\triangle CMO \cong \triangle CNO$ and it follows that $\angle COM = \angle CON$. But $\angle MON = 60°$. Therefore, $\angle COM = \angle CON = 30°$.

In $\triangle CMO$, $CM = CO = r$ and $\triangle CMO$ is isosceles. Therefore, $\angle CMO = \angle COM = 30°$ and $\angle MCO = 180° - 30° - 30° = 120°$.

Method 1: Using the sine law,

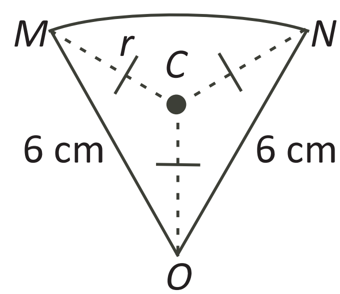
$$\begin{aligned}
\frac{CM}{\sin(\angle COM)} &= \frac{OM}{\sin(\angle MCO)} \\
\frac{r}{\sin 30°} &= \frac{6}{\sin 120°} \\
r &= \frac{6}{\sin 120°} \times \sin 30° \\
r &= \frac{6}{\frac{\sqrt{3}}{2}} \times \frac{1}{2} \\
r &= 6 \times \frac{2}{\sqrt{3}} \times \frac{1}{2} \\
r &= \frac{6}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \\
r &= 2\sqrt{3} \text{ cm}
\end{aligned}$$



The radius of the circle that passes through M, N, and O is $2\sqrt{3}$ cm.

Method 2: Using the cosine law,

$$\begin{aligned}
CM^2 &= CO^2 + MO^2 - 2 \times CO \times MO \times \cos(\angle COM) \\
r^2 &= r^2 + 6^2 - 2(6)(r)\cos 30^\circ \\
12r\cos 30^\circ &= 36 \\
r\cos 30^\circ &= 3 \\
r \times \frac{\sqrt{3}}{2} &= 3 \\
r \times \sqrt{3} &= 6 \\
r \times \sqrt{3} \times \sqrt{3} &= 6 \times \sqrt{3} \\
3r &= 6\sqrt{3} \\
r &= 2\sqrt{3} \text{ cm}
\end{aligned}$$



The radius of the circle that passes through M, N, and O is $2\sqrt{3}$ cm.