



Problem of the Week

Problem E and Solution

Lucky Lucky

Problem

A bag contains ten identical balls, each numbered with a different number from the integers 1 to 10. Lucky Pix draws three balls from the bag and holds them in her hand. She wins if the smallest numbered ball in her hand is odd and the next smallest numbered ball in her hand is even. Determine the probability that Lucky wins.

Solution

To determine the probability we need to determine two things: the total number of different three-ball selections and the total number of winning selections.

First, we will determine the total number of different three-ball selections. Since each number is distinct, then there are 10 choices for the first ball, 9 choices for the second ball and 8 choices for the third ball. This produces $10 \times 9 \times 8 = 720$ ordered selections. But this total includes 6 orderings for each possible selection of three numbers. For example, the three numbers 1, 2, and 3, would be included 6 times: (1,2,3), (1,3,2), (2,1,3), (2,3,1), (3,1,2), and (3,2,1). Basically, each three-ball selection is counted six times. Therefore, there are $720 \div 6 = 120$ different possible three-ball selections.

Next, we will determine the number of winning three-ball selections. That is, the number of selections in which the smallest number is odd and the next smallest number is even. The information is presented in chart form.

Smallest Number	Second Smallest Number	Possible Value(s) for the Largest Number	Number of Three-ball Selections
1	2	3,4,5,6,7,8,9,10	8
	4	5,6,7,8,9,10	6
	6	7,8,9,10	4
	8	9,10	2
3	4	5,6,7,8,9,10	6
	6	7,8,9,10	4
	8	9,10	2
5	6	7,8,9,10	4
	8	9,10	2
7	8	9,10	2

The total number of winning selections is

$$(8 + 6 + 4 + 2) + (6 + 4 + 2) + (4 + 2) + (2) = 20 + 12 + 6 + 2 = 40.$$

The probability that Lucky wins is $\frac{40}{120} = \frac{1}{3}$.

