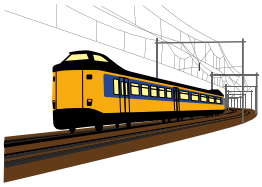

Problem of the Week
Problem E and Solution
Keep On Chuggin'

Problem

Two trains of equal length are on parallel tracks. One train is travelling at 40 km/h and the other at 20 km/h. It takes two minutes longer for the trains to completely pass one another when going in the same direction, than when going in opposite directions. Determine the length of each train.

Solution

Solution 1

Let L represent the length, in km, of each train. Let t_1 represent the time, in hours, required for the fast train to completely pass the slow train when going in the same direction. Let t_2 represent the time, in hours, required for the fast train to completely pass the slow train when going in opposite directions.

In order to completely pass one another when going in the same direction, the faster train must travel two lengths of the train plus whatever distance the slower train travels. Therefore,

$$\begin{aligned} 40t_1 &= 20t_1 + 2L \\ 20t_1 &= 2L \\ t_1 &= \frac{L}{10} \end{aligned}$$

In order to completely pass one another when going in the opposite direction, the total distance travelled by the two trains must be $2L$. Therefore,

$$\begin{aligned} 40t_2 + 20t_2 &= 2L \\ 60t_2 &= 2L \\ t_2 &= \frac{L}{30} \end{aligned}$$

Since it takes two minutes or $\frac{2}{60}$ hours longer for the trains to completely pass one another when going in the same direction than when going in opposite directions,

$$\begin{aligned} t_1 - t_2 &= \frac{2}{60} \\ \frac{L}{10} - \frac{L}{30} &= \frac{1}{30} \\ \text{Multiplying by 30:} \quad 3L - L &= 1 \\ 2L &= 1 \\ L &= 0.5 \end{aligned}$$

Therefore, the length of each train is 0.5 km.

Solution 2

Let L represent the length, in km, of each train.

When going in the same direction, the faster train is travelling at $40 - 20 = 20$ km/h relative to the slower train. In order to completely pass, the faster train must travel $2L$ km. Therefore, it takes $\frac{2L}{20} = \frac{L}{10}$ hours to completely pass.

When travelling in opposite directions, the faster train is travelling at $40 + 20 = 60$ km/h relative to the slower train. In order to completely pass, the faster train must travel $2L$ km. Therefore, it takes $\frac{2L}{60} = \frac{L}{30}$ hours to completely pass.

Since it takes two minutes or $\frac{2}{60} = \frac{1}{30}$ hours longer for the trains to completely pass one another when going in the same direction than when going in opposite directions,

$$\frac{L}{10} - \frac{L}{30} = \frac{1}{30}$$

Multiplying by 30:

$$\begin{aligned} 3L - L &= 1 \\ 2L &= 1 \\ L &= 0.5 \end{aligned}$$

Therefore, the length of each train is 0.5 km.

Solution 3

Let L represent the length, in km, of each train.

While the trains are travelling in opposite directions, let y km be the distance travelled by the slower train from the time the faster train begins to pass until it completely passes. The slower train travels y km and the faster train travels $(2L - y)$ km. We know that the time travelled will be the same so:

$$\begin{aligned} \frac{y}{20} &= \frac{2L-y}{40} \\ \frac{2y}{40} &= \frac{2L-y}{40} \\ 3y &= 2L \qquad (1) \end{aligned}$$

While the trains are travelling in the same direction, let x km be the distance travelled by the slower train from the time the faster train begins to pass until it completely passes. The slower train travels x km and the faster train travels $(x + 2L)$ km. We know that the time travelled will be the same so:

$$\begin{aligned} \frac{x}{20} &= \frac{x+2L}{40} \\ \frac{2x}{40} &= \frac{x+2L}{40} \\ x &= 2L \qquad (2) \end{aligned}$$

We know that it takes two minutes or $\frac{2}{60}$ hours longer for the trains to completely pass one another when going in the same direction than when going in opposite directions. So,

$$\begin{aligned} \frac{x}{20} - \frac{y}{20} &= \frac{2}{60} \\ \frac{x}{20} - \frac{3y}{60} &= \frac{2}{60} \end{aligned}$$

Substituting $2L$ for x from (2) and $2L$ for $3y$ from (1),

$$\begin{aligned} \frac{2L}{20} - \frac{2L}{60} &= \frac{2}{60} \\ \frac{6L}{60} - \frac{2L}{60} &= \frac{2}{60} \\ 4L &= 2 \\ L &= 0.5 \end{aligned}$$

Therefore, the length of each train is 0.5 km.