



## Problem of the Week

### Problem E and Solution

#### Above Average Task

#### Problem

Four numbers are selected such that when each number is added to the average of the other three, the following sums are obtained: 25, 37, 43, and 51.

Determine the average of the four numbers.

#### Solution

It is possible to precisely determine the four numbers but the problem only asks for their average. Let  $a, b, c, d$  represent each of the four numbers. We are looking for  $\frac{a+b+c+d}{4}$ .

When the first number is added to the average of the other three numbers the result is 25.

$$\therefore a + \frac{b+c+d}{3} = 25 \text{ which simplifies to } 3a + b + c + d = 75 \quad (1)$$

When the second number is added to the average of the other three numbers the result is 37.

$$\therefore b + \frac{a+c+d}{3} = 37 \text{ which simplifies to } a + 3b + c + d = 111 \quad (2)$$

When the third number is added to the average of the other three numbers the result is 43.

$$\therefore c + \frac{a+b+d}{3} = 43 \text{ which simplifies to } a + b + 3c + d = 129 \quad (3)$$

When the fourth number is added to the average of the other three numbers the result is 51.

$$\therefore d + \frac{a+b+c}{3} = 51 \text{ which simplifies to } a + b + c + 3d = 153 \quad (4)$$

Adding (1), (2), (3), and (4) we obtain  $6a + 6b + 6c + 6d = 468$ . Dividing by 6,  $a + b + c + d = 78$ . So the sum of the four numbers is 78. Dividing by 4 we determine that the average of the four numbers is 19.5.

Therefore the average of the four numbers is 19.5.

(By solving the system of equations we can actually determine that the numbers are:  $-1.5, 16.5, 25.5,$  and  $37.5$ .) The possible T-shirt logo represents “Well Above Average”, something worth striving for.

