



Problem of the Week

Problem E and Solution

Ahead of It's Time

Problem

When Jeff received a pocket watch from his Grandmother on his 12th birthday it was set at precisely the correct time. However, Jeff soon discovered that his watch gained exactly 10 seconds every day. Assuming that Jeff never adjusts his watch to correct the time, how many times after his 12th birthday and before his 90th birthday will his watch show the correct time?

Solution

Solving this problem is not difficult. However, the answer may surprise the solver.

The watch will be correct once it has gained 12 hours.

$$12 \text{ h} = 12 \times 60 = 720 \text{ minutes}$$

$$720 \text{ minutes} = 720 \times 60 = 43\,200 \text{ seconds}$$

Since the watch gains 10 seconds every day, it will take $43\,200 \div 10 = 4\,320$ days or approximately $4\,320 \div 365 = 11.8$ years until it is the correct time again.

From Jeff's 12th birthday to his 90th birthday, 78 years pass. The watch will be accurate $78 \div (4\,320 \div 365) \doteq 6.6$ times. This means his watch will be accurate only 6 times after his 12th birthday and before his 90th birthday.

The watch will be correct when he is 23.8 years old (between his 23rd and 24th birthday), when he is 35.7 years old (between his 35th and 36th birthday), when he is 47.5 years old (between his 47th and 48th birthday), when he is 59.3 years old (between his 59th and 60th birthday), when he is 71.2 years old (near his 71st birthday), and when he is 83.0 years old (near his 83rd birthday). Jeff may wish to correct his watch periodically or get a more accurate one.

As a concluding note, if the watch gained one second per day, the watch would never be correct again for approximately 120 years. The answer is surprising!

