Problem of the Week
Problem E and Solution
Marvel at This

Problem

With the renewed interest in movies based on comic book characters, many clubs for comic book collectors have started. One such club attracts between 15 and 35 members to their monthly meetings. At their last meeting, they discovered that all of the members in attendance had exactly the same number of comic books, except for one member who had one more comic book than each of the other members. Between them, the members had precisely 1 000 comic books. How many members attended the last meeting?

Solution

One could attempt a trial and error solution to this problem. However, a more algebraic solution will be presented here.

Let n represent the number of members present at the last monthly meeting such that $15 < n < 35$ and n is an integer. Let c represent the number of comic books that all but one member had. The one member had $c + 1$ comic books. It follows that $(n - 1)$ members had c comic books each and one member had $c + 1$ comic books producing a total of 1000 comic books.

$$\begin{aligned}(n-1)c + 1(c+1) &= 1000\\ nc - c + c + 1 &= 1000\\ nc &= 999\end{aligned}$$

We are looking for two positive integers with a product of 999 with one of the numbers between 15 and 35. The prime factorization of 999 is $3 \times 3 \times 3 \times 37$. We can combine the factors to produce pairs of positive integers whose product is 999. The possibilities are 1 and 999, 3 and 333, 9 and 111, and 27 and 37. The only possible product which gives one factor between 15 and 35 is 27×37.

It then follows that there were 27 members present at the last meeting, 26 of the members had 37 comic books each and 1 member had 38 comic books. (This is easily verified: $26 \times 37 + 1 \times 38 = 1000$.)