



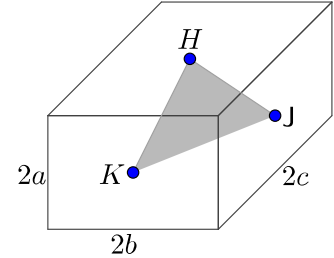
Problem of the Week

Problem E and Solution

Volumizing A Triangle

Problem

A rectangular prism has dimensions of $2a$, $2b$ and $2c$. H is the intersection of the diagonals of the top face of the prism, J is the intersection of the diagonals of the side face of the prism and K is the intersection of the diagonals of the front face of the prism. $\triangle HJK$ is formed by joining H , J and K . This is shown in the diagram to the right. If $HJ = 4$ cm, $HK = 5$ cm, and $JK = 6$ cm, determine the volume of the rectangular prism.



Solution

Label the top front edge of the rectangular prism AB and its midpoint M . Then draw in $\triangle KMH$.

Since H and K are the centres of their respective rectangles, $MK = a$, $MH = c$ and $HK = 5$.

Since $\angle KMH = 90^\circ$, then
$$a^2 + c^2 = 25 \quad (1)$$

Similarly, it can be shown that
$$b^2 + c^2 = 36 \quad (2)$$

and
$$a^2 + b^2 = 16 \quad (3)$$

Adding (1), (2), and (3),
$$2a^2 + 2b^2 + 2c^2 = 77$$

Then dividing by 2,
$$a^2 + b^2 + c^2 = \frac{77}{2} \quad (4)$$

Subtracting each of equations (1), (2) and (3) from equation (4) yields

$$b^2 = \frac{27}{2}, \quad a^2 = \frac{5}{2}, \quad \text{and} \quad c^2 = \frac{45}{2}.$$

Multiplying a^2 , b^2 and c^2 gives the product $a^2b^2c^2 = \frac{(5)(27)(45)}{8} = \frac{6075}{8}$.

Then, taking the positive square root, $abc = \sqrt{\frac{6075}{8}} = \frac{45\sqrt{3}}{2\sqrt{2}} = \frac{45\sqrt{3}}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{45\sqrt{6}}{4}$.

To determine the volume of the rectangular prism multiply the side lengths $2a$, $2b$ and $2c$ to obtain $V = (2a)(2b)(2c) = 8abc = 8 \left(\frac{45\sqrt{6}}{4} \right) = (2\sqrt{2})(45\sqrt{3}) = 90\sqrt{6} \text{ cm}^3$.

The volume of the rectangular prism is $90\sqrt{6} \text{ cm}^3$.

