Problem of the Week
Problem E and Solution
How Far to the Centre

Problem

A circle with centre O is drawn with points P, Q, and S on the circumference such that $PQ = PS = 12$ m. PO is extended to meet QS at R such that $PR \perp QS$ and $OR = 1$ m.

Determine the radius of the circle.

Solution

Since O is the centre of a circle that passes through P, Q, and S, then OP, OQ, and OS are radii. Then $OP = OQ = OS = x$, $x > 0$. Let $SR = y$.

$\triangle SPR$ is right angled at R. Using the Pythagorean Theorem,

$$
\begin{aligned}
PR^2 + RS^2 &= PS^2 \\
(PO + OR)^2 + RS^2 &= PS^2 \\
(x+1)^2 + y^2 &= 12^2 \qquad (1)
\end{aligned}
$$

$\triangle SOR$ is right angled at R. Using the Pythagorean Theorem,

$$
\begin{aligned}
OR^2 + RS^2 &= OS^2 \\
1^2 + y^2 &= x^2 \\
y^2 &= x^2 - 1
\end{aligned}
$$

Substitute for y^2 in (1):

$$
\begin{aligned}
(x+1)^2 + x^2 - 1 &= 12^2 \\
x^2 + 2x + 1 + x^2 - 1 &= 144 \\
2x^2 + 2x - 144 &= 0 \\
x^2 + x - 72 &= 0 \\
(x-8)(x+9) &= 0 \\
x = 8 \quad \text{or} \quad x &= -9
\end{aligned}
$$

Since $x > 0$, $x = -9$ is inadmissible. Therefore, $x = 8$. But x is the radius of the circle.

$\therefore$ the radius of the circle is 8 m.