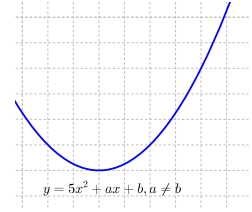




## Problem of the Week

### Problem E and Solution

#### How Low Will It Go?



#### Problem

Suppose  $y = 5x^2 + ax + b$ ,  $a \neq b$ , is a parabola that passes through the points  $A(a, b)$  and  $B(b, a)$ . Determine the minimum value of the parabola.

#### Solution

Since  $A(a, b)$  is on the parabola, it satisfies the equation of the parabola. We can substitute  $x = a$  and  $y = b$  into the equation  $y = 5x^2 + ax + b$ .

$$b = 5a^2 + a^2 + b$$

$$b = 6a^2 + b$$

$$0 = 6a^2$$

$$0 = a^2$$

$$0 = a$$

The equation becomes  $y = 5x^2 + 0x + b$  or simply  $y = 5x^2 + b$ .

Since  $B(b, a)$  is on the parabola, it satisfies the equation of the parabola. We can substitute  $x = b$  and  $y = a = 0$  into the equation  $y = 5x^2 + b$ .

$$0 = 5b^2 + b$$

$$0 = b(5b + 1)$$

$$b = 0 \quad \text{or} \quad 5b + 1 = 0$$

$$b = -\frac{1}{5}$$

Since  $a \neq b$  and  $a = 0$ , then  $b = 0$  is inadmissible. Therefore,  $b = -\frac{1}{5}$  and the equation becomes  $y = 5x^2 - \frac{1}{5}$ . The vertex of the parabola is  $(0, -\frac{1}{5})$  and so the minimum value is  $-\frac{1}{5}$ .

