



Problem of the Week

Problem E and Solution

Every Vote Counts!

Problem

In a recent election, the ratio of the number of voters for the Purple Party to the number of voters for the Pink Party was 15:16 and the Pink Party won the election. Had 300 more people voted for the Purple Party and 200 fewer people voted for the Pink Party, the ratio would have been 11:10 and the Purple Party would have won the election. Determine the total number of votes originally cast.

Solution

Let a represent the number of votes originally cast for the Purple Party.

Let b represent the number of votes originally cast for the Pink Party.

Then the total number of votes originally cast was $a + b$.

The original ratio of votes cast was $a : b = 15 : 16$. This ratio can be written $\frac{a}{b} = \frac{15}{16}$ and $a = \frac{15}{16}b$ follows. (1)

Had 300 more people voted for the Purple Party, the Purple Party would have received $(a + 300)$ votes. Had 200 fewer people voted for the Pink Party, the Pink Party would have received $(b - 200)$ votes. Then

$$\frac{a + 300}{b - 200} = \frac{11}{10}$$

$$10a + 3000 = 11b - 2200$$

$$10a = 11b - 5200$$

$$10 \left(\frac{15b}{16} \right) = 11b - 5200 \quad \text{Substituting for } a \text{ from (1)}$$

$$5 \left(\frac{15b}{8} \right) = 11b - 5200$$

$$75b = 88b - 41600 \quad \text{Multiplying by 8}$$

$$-13b = -41600$$

$$b = 3200$$

$$a = \frac{15}{16}(3200) \quad \text{Substituting } b = 3200 \text{ in (1)}$$

$$a = 3000$$

$$a + b = 3000 + 3200$$

$$a + b = 6200$$

There were 6 200 total votes originally cast.

VOTE

VOTE

