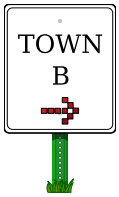
Problem of the Week
Problem D and Solution
Keep on Trackin'



Problem

A freight train is 27 minutes late when it makes its usual trip between Town A and Town B at an average speed of 56 km/h. For another trip between Town A and Town B, the freight train is 42 minutes late when its average speed is 54 km/h. What is the distance between Town A and Town B?

Solution

Solution 1

Let t represent the time, in hours, taken by the train when it was 27 minutes late. Then, $t + \frac{1}{4}$ represents the time, in hours, taken by the train when it was 15 minutes or one-quarter hour later.

The following chart displays the information. The distance column of the chart is completed by multiplying the average speed by the time.

	Distance	Speed	Time
Faster Train	$56t$	56	t
Slower Train	$54(t + \frac{1}{4})$	54	$t + \frac{1}{4}$

Since the distance between Town A and Town B remains constant,

$$\begin{aligned} 56t &= 54(t + \frac{1}{4}) \\ 56t &= 54t + \frac{27}{2} \\ 2t &= \frac{27}{2} \\ t &= \frac{27}{4} \end{aligned}$$

The distance between Town A and Town B is $56t = 56 \times \frac{27}{4} = 378$ km.

Solution 2

Let d represent the distance, in km, between Town A and Town B.

The following chart displays the information. The time column of the chart is completed by dividing the distance by the average speed.

	Distance	Speed	Time
Faster Train	d	56	$\frac{d}{56}$
Slower Train	d	54	$\frac{d}{54}$

Since the difference in times between the slower train and the faster train is 15 minutes or $\frac{1}{4}$ hour,

$$\begin{aligned} \frac{d}{54} - \frac{d}{56} &= \frac{1}{4} \\ \frac{56d - 54d}{(54)(56)} &= \frac{1}{4} \\ 2d &= \frac{1}{4} \times (54)(56) \\ 2d &= 756 \\ d &= 378 \end{aligned}$$

The distance between Town A and Town B is 378 km.

Solution 3

This solution looks at the problem quite differently from the first two solutions.

If the faster train first travels for 27 minutes, it would then complete the rest of the trip in the usual amount of time. During the 27 minutes, the faster train would travel $\frac{27}{60} \times 56 = \frac{1512}{60} = 25.2$ km.

If the slower train first travels for 42 minutes, it would then complete the rest of the trip in the usual amount of time. During the 42 minutes, the slower train would travel $\frac{42}{60} \times 54 = \frac{2268}{60} = 37.8$ km.

The slower train is $37.8 - 25.2 = 12.6$ km ahead of the faster train at the point when the usual time to complete the trip remains. The faster train gains 2 km/h on the slower train. It will take the faster train $\frac{12.6}{2} = 6.3$ h to catch up and thereby complete the trip. In 6.3 h, the faster train travels $6.3 \times 56 = 352.8$ km. But it had already travelled 25.2 km. Therefore the total distance from Town A to Town B is $25.2 + 352.8 = 378$ km.

The distance between Town A and Town B is 378 km.