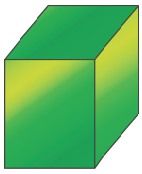
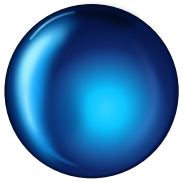


Problem of the Week
Problem D and Solution
Weight and See



Problem

Icon Duit was asked to determine the combined weight of one sphere and one cube but he did not have a reliable weigh scale. He was, however, given the following information: four spheres and three cubes weigh 37 g, and three spheres and four cubes weigh 33 g. Icon was quickly able to determine the combined weight, in grams, of one sphere and one cube. Your task is to also determine the combined weight of one sphere and one cube.

Solution

Solution 1

The problem is solved quickly if you make the following observation. Since four spheres and three cubes weigh 37 g and three spheres and four cubes weigh 33 g, then, by combining the two pieces of information, seven spheres and seven cubes weigh $37 + 33 = 70$ g. Dividing by 7, one sphere and one cube weigh $70 \div 7 = 10$ g.

Solution 2

In this solution we will find the weight of one sphere and the weight of one cube. We will then determine the sum.

Let c represent the weight, in grams, of one cube.
Let s represent the weight, in grams, of one sphere.

From the first piece of information, $4s + 3c = 37$. (1)
From the second piece of information, $3s + 4c = 33$. (2)

To get from equation (1) to equation (2), notice that there is one less sphere and one more cube causing the weight to go down 4 g. Therefore, a sphere weighs 4 g more than a cube and $s = c + 4$ follows. We can substitute for s in (1).

$$\begin{aligned} 4(c+4) + 3c &= 37 \\ 4c + 16 + 3c &= 37 \\ 7c &= 21 \\ c &= 3 \text{ g} \end{aligned}$$

Since $c = 3$ and $s = c + 4$, $s = 7$ g. One sphere weighs 7 g and one cube weighs 3 g. The combined weight of one sphere and one cube is $3 + 7 = 10$ g.

Solution 3

Let c represent the weight, in grams, of one cube.
Let s represent the weight, in grams, of one sphere.

Using the given information, we obtain the following system of equations.

$$\begin{aligned} 4s + 3c &= 37 \qquad (1) \\ 3s + 4c &= 33 \qquad (2) \end{aligned}$$

We will now use elimination to solve for s and c.

$$\begin{aligned} \text{Multiply (1) by 4} \qquad 16s + 12c &= 148 \qquad (3) \\ \text{Multiply (2) by 3} \qquad 9s + 12c &= 99 \qquad (4) \end{aligned}$$

$$\begin{aligned} \text{Subtracting (4) from (3)} \qquad 7s &= 49 \\ s &= 7 \text{ g} \end{aligned}$$

$$\begin{aligned} \text{Substituting } s = 7 \text{ into (1)} \qquad 4(7) + 3c &= 37 \\ 28 + 3c &= 37 \\ 3c &= 9 \\ c &= 3 \text{ g} \end{aligned}$$

Therefore, one sphere weighs 7 g and one cube weighs 3 g. The combined weight of one sphere and one cube is $3 + 7 = 10$ g.

Solution 4

This is the algebraic version of solution 1.

Let c represent the weight, in grams, of one cube.
Let s represent the weight, in grams, of one sphere.

Using the given information, we obtain the following system of equations.

$$\begin{aligned} 4s + 3c &= 37 \qquad (1) \\ 3s + 4c &= 33 \qquad (2) \end{aligned}$$

Adding (1) and (2), we obtain $7s + 7c = 70$. Dividing both sides of the equation by 7, we obtain $s + c = 10$. But s is the weight of one sphere and c is the weight of one cube so $s + c$ is the combined weight of one sphere and one cube.

Therefore, the combined weight of one sphere and one cube is 10 g.