



Problem of the Week

Problem D and Solution

Get a New Clock, Really!

Problem

Dante B Late has a clock that gains exactly 12 minutes per hour. Just before leaving for work at 5:30 a.m., Dante set the clock to the correct time. Dante returned home for lunch and while eating glanced over at the clock. The clock read 1:30 p.m. At first Dante was concerned, knowing that he should have left the house at 12:30 p.m. in order to return to work on time. But then he remembered the clock's peculiar feature. How much time does Dante have left at home before he needs to leave to return to work?

Solution

Solution 1

Every hour the clock gains 12 minutes or $\frac{12}{60} = \frac{1}{5}$ h.

Let x be the actual number of hours that have passed since 5:30 a.m. Then the number of hours that the clock advances is $x + \frac{1}{5}x = \frac{6}{5}x$ h.

From 5:30 a.m. to 1:30 p.m., the clock advances 8 h.

$$\therefore \frac{6}{5}x = 8$$

$$x = \frac{20}{3}$$

$$x = 6\frac{2}{3} \text{ h}$$

$$x = 6 \text{ h } 40 \text{ min}$$

The actual time is 6 hours and 40 minutes after 5:30 a.m. This puts the actual time at 12:10 p.m. and Dante still has 20 minutes left before he must leave to return to work.



**Solution 2**

For every 60 real minutes, the clock advances $60 + 12 = 72$ minutes.

From 5:30 a.m. to 1:30 p.m. the clock advances 8 h or $8 \times 60 = 480$ minutes.

Let x be the number of real minutes that pass when the clock has advanced 480 minutes.

$$\text{Then } \frac{x}{480} = \frac{60}{72}$$

$$\frac{x}{480} = \frac{5}{6}$$

$$x = \frac{5}{6}(480)$$

$$x = 400 \text{ min}$$

$$x = 6 \text{ h } 40 \text{ min}$$

The real time is 6 hours and 40 minutes after 5:30 a.m. or 12:10 p.m. Dante still has 20 minutes left before he needs to leave to return to work.

Solution 3

Actual Number of Hours	Real Time	Clock Advances	Clock Time
	5:30 a.m.		5:30 a.m.
1	6:30 a.m.	1 h 12 min	6:42 a.m.
2	7:30 a.m.	2 h 24 min	7:54 a.m.
3	8:30 a.m.	3 h 36 min	9:06 a.m.
4	9:30 a.m.	4 h 48 min	10:18 a.m.
5	10:30 a.m.	6 h 0 min	11:30 a.m.

When the clock advances 6 h from 5:30 a.m to 11:30 a.m., 5 real hours have passed. So how much real time passes as the clock advances from 11:30 a.m. to 1:30 p.m.? We know that when the clock advances 6 hours, 5 real hours pass. Dividing each of these numbers by 3, we determine that when the clock advances 2 h, $\frac{5}{3}$ or $1\frac{2}{3}$ real hours elapse. When the clock moves from 11:30 a.m. to 1:30 p.m., $1\frac{2}{3}$ h or 1 hour and 40 minutes of real time will pass from 10:30 a.m. actual time.

The actual time is then 12:10 p.m. and Dante can relax. He still has 20 minutes before he must leave to return to work.

