



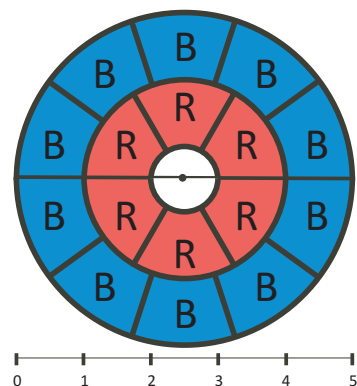
Problem of the Week

Problem D and Solution

A Colourful View

Problem

A large stained glass window is designed using semicircles. The diameter of the largest semicircle is 5 m, the diameter of the middle semicircle is 3 m, and the diameter of the smallest semicircle is 1 m. All of the blue areas, marked B , are equal and all of the red areas, marked R , are equal. Determine the ratio of one area marked R to one area marked B .



Solution

Make a copy of the window and flip it over. Attach the two halves at the diameter creating a circular window. The resulting window is shown above. The ratio of area R to area B will remain unchanged. This construction is not necessary but it allows us to find the area of circles rather than semicircles.

The smallest circle has diameter 1 m and radius $\frac{1}{2}$ m. The area of the small circle is $\pi\left(\frac{1}{2}\right)^2 = \frac{1}{4}\pi$ m².

The middle circle has diameter 3 m and radius $\frac{3}{2}$ m. The area of the middle circle is $\pi\left(\frac{3}{2}\right)^2 = \frac{9}{4}\pi$ m². But the area of the middle circle is made up of six areas marked R plus the small inner circle. Therefore,

$$\begin{aligned} 6R + \frac{1}{4}\pi &= \frac{9}{4}\pi \\ 6R &= \frac{9}{4}\pi - \frac{1}{4}\pi \\ 6R &= \frac{8}{4}\pi \\ 6R &= 2\pi \\ R &= \frac{\pi}{3} \text{ m}^2 \end{aligned}$$





The large circle has diameter 5 m and radius $\frac{5}{2}$ m. The area of the large circle is $\pi(\frac{5}{2})^2 = \frac{25}{4}\pi$ m². But the area of the large circle is made up of ten areas marked B plus the area of the middle circle calculated earlier. Therefore,

$$\begin{aligned}10B + \frac{9}{4}\pi &= \frac{25}{4}\pi \\10B &= \frac{25}{4}\pi - \frac{9}{4}\pi \\10B &= \frac{16}{4}\pi \\10B &= 4\pi \\B &= \frac{2\pi}{5} \text{ m}^2\end{aligned}$$

The ratio of one red area R to one blue area B is now easily calculated.

$$\begin{aligned}R : B &= \frac{\pi}{3} : \frac{2\pi}{5} \\&= \frac{5\pi}{15} : \frac{6\pi}{15} \\&= 5 : 6\end{aligned}$$

The ratio of area R to area B is 5 : 6. In other words, one area R is five-sixths of one area B .

