



Problem of the Week

Problem D and Solution

A Good Use for Pennies

Problem

Alice and Beth are playing a game with three pennies on a 1 unit by 8 unit strip of paper divided up into eight squares. To start the game, the pennies are placed in the three leftmost positions, as shown. The rules of the game are as follows:

- (i) On a player's turn, the player must move a penny any number of squares to the right.
- (ii) The penny may not pass over any other penny or land on a square that is occupied by another penny.
- (iii) If it is a player's turn and there is no legal move possible, then that player loses the game.

It is Alice's turn to go first. Alice can always win. Describe Alice's first move and her winning strategy.

Solution

Consider playing the game with just two pennies and four squares. We have numbered the squares for easier reference.

We will use P to represent a square that contains a penny and E to represent an empty square.

①	②	③	④
P	P	E	E

Player 1 can only move the penny in box ② to either box ④ or box ③.

- If Player 1 moves the penny in box ② two squares to the right to box ④,

①	②	③	④
P	E	E	P

then Player 2 would be able to win the game by moving the penny in box ① two squares to the right to box ③.

①	②	③	④
E	E	P	P

So, if Player 1 moves the penny in box ② two squares to the right to box ④ then Player 2 should win the game.





- If Player 1 moves the penny in box ② one square to the right to box ③,

①	②	③	④
P	E	P	E

then Player 2 should not move the penny in box ③ one square to the right to box ④ because Player 1 could then win the game on the following turn by moving the penny in box ① two squares to the right to box ③. This is illustrated below.

①	②	③	④
P	E	E	P
E	E	P	P

So Player 2 should move the penny in box ① one square to the right to box ②.

①	②	③	④
E	P	P	E

Player 1 is forced to move the penny in box ③ one square to the right to box ④.

①	②	③	④
E	P	E	P

On the following turn, Player 2 wins the game by moving the penny in box ② one square to the right to box ③.

①	②	③	④
E	E	P	P

So, if Player 1 moves the penny in box ② one square to the right to box ③ then Player 2 should win the game.

In our game with just two pennies and four squares, Player 2 is always able to win. If you look closely you will see that whatever Player 1 did, Player 2 copied with the other coin. The two pennies start together. Player 1 makes a move to the right creating a gap between the two pennies. On the following turn, Player 2 moves the other penny in such a way that there is no space between the two pennies. The number of squares really does not matter. Whatever Player 1 does with the coin on the right, Player 2 “mimics” with the coin on the left. But Player 2 wins in this version of the game. In our game Player 1, Alice, is supposed to win.

Alice basically needs to make a first move so that what is left after her move is the same as our two penny game.

①	②	③	④	⑤	⑥	⑦	⑧
P	P	P	E	E	E	E	E

To do this, Alice should moves the penny in box ③ to the far right to box ⑧.

①	②	③	④	⑤	⑥	⑦	⑧
P	P	E	E	E	E	E	P

This creates the two penny game with a total of 7 boxes. Now, whatever Beth does on her turn with the penny that starts in box ②, Alice should “mimic” with the penny that starts in box ① and she will win the game.

