



Problem of the Week

Problem D and Solution

Not That Obvious

Problem

	Column 1	Column 2	Column 3	Column 4
Row 1	51	52	a	54
Row 2	b	56	57	c
Row 3	59	d	61	e

The sum of the numbers in each of the three rows is the same. The sum of the numbers in each of the four columns is the same. However, the sum of any row does not equal the sum of any column. Determine the missing values in the table.

Solution

Each of the first three columns has two known values and one unknown value. We also know that the sum of each column is the same.

$$\begin{array}{lcl}
 \text{Sum of Column 2} = \text{Sum of Column 1} & \text{and} & \text{Sum of Column 3} = \text{Sum of Column 1} \\
 52 + 56 + d = 51 + b + 59 & & a + 57 + 61 = 51 + b + 59 \\
 d + 108 = b + 110 & & a + 118 = b + 110 \\
 \therefore d = b + 2 & & \therefore a = b - 8
 \end{array}$$

The table can now be redrawn with a few less variables.

	Column 1	Column 2	Column 3	Column 4
Row 1	51	52	$b - 8$	54
Row 2	b	56	57	c
Row 3	59	$b + 2$	61	e

The table will be reproduced at the top of the next page.





	Column 1	Column 2	Column 3	Column 4
Row 1	51	52	$b - 8$	54
Row 2	b	56	57	c
Row 3	59	$b + 2$	61	e

We also know that the sum of the numbers in each row is the same.

$$\begin{aligned} \text{Sum of Row 2} &= \text{Sum of Row 1} \\ b + 56 + 57 + c &= 51 + 52 + (b - 8) + 54 \\ b + c + 113 &= b + 149 \\ c &= 36 \end{aligned}$$

We can determine the value of e by equating the sum of row 3 and the sum of row 1.

$$\begin{aligned} \text{Sum of Row 3} &= \text{Sum of Row 1} \\ 59 + (b + 2) + 61 + e &= 51 + 52 + (b - 8) + 54 \\ b + e + 122 &= b + 149 \\ e &= 27 \end{aligned}$$



Since we know $c = 36$ and $e = 27$, we can determine the column sum using the fourth column. The column sum is $54 + c + e = 54 + 36 + 27 = 117$. We can use the column sum with the first column to determine the value of b . Then $51 + b + 59 = 117$ and $b = 7$ follows. We know that $d = b + 2$ so $d = 9$. Also, we know that $a = b - 8$ so $a = -1$.

The missing values in the table are $a = -1$, $b = 7$, $c = 36$, $d = 9$, and $e = 27$. The completed table follows. We can use it to verify that each row sums to 156 and each column sums to 117.

	Column 1	Column 2	Column 3	Column 4	Row Sum
Row 1	51	52	-1	54	156
Row 2	7	56	57	36	156
Row 3	59	9	61	27	156
Column Sum	117	117	117	117	

