



$$B > \frac{1}{n} \sum_{i=1}^n x_i$$

Problem of the Week

Problem D and Solution

Just Average

Problem

Four of the six numbers 1867, 1994, 2028, 2034, 2126, and 2139 have a mean (average) of 2017. Determine the mean (average) of the other two numbers from the list.

Solution

At the outset, it should be noted that we could “play” with the numbers to determine which of the four numbers have an average of 2017. We could then easily determine the average of the remaining two numbers. This method works decently well on a problem with a small number of numbers. However, if we were to increase the size of the list by just a few more numbers, then the task would not easily be solved using this approach. For example, if the list contained 100 numbers and the mean of 20 of them was 2017, solving the problem by playing would not be easy.

The sum of the six numbers in the list is

$$1867 + 1994 + 2028 + 2034 + 2126 + 2139 = 12\,188.$$

Since the average of four of the numbers is 2017, then the sum of those four numbers is $4 \times 2017 = 8068$. (We do not necessarily know which four numbers produce this sum, but we do not actually need to know.)

The sum of the two remaining integers is $12\,188 - 8068 = 4120$. Since there are two integers in the sum, the average is easily calculated by dividing the sum by 2. The average of the remaining two integers is then $4120 \div 2 = 2060$.

Although not required, the two integers from the list that sum to 4120 are 1994 and 2126. It is then easily verified that the average of the four other integers, 1867, 2028, 2034, and 2139, is 2017.

The Meaning of the Graphic:

The notation $\frac{1}{n} \sum_{i=1}^n x_i$ is a mathematical short form which represents the average

of the n numbers x_1, x_2, \dots, x_n . So the logo on the T-shirt is challenging you to “Be more than average.”

