



Problem of the Week

Problem D and Solution

E Z Does It

Problem

E Z Dealer has a deck consisting of 100 cards numbered from 1 to 100. Each card has the same number printed on both sides. One side of the card is yellow and the other side of the card is red. E Z places all the cards, red side up, on the table. He first turns over every card that has a number which is a multiple of 2. He then examines all the cards, and turns over every card that has a number which is a multiple of 3. After E Z has finished, how many cards have the red side facing up?

Solution

After flipping over all of the cards with numbers that are multiples of 2, 50 red cards are facing up and 50 yellow cards are facing up. All of the red cards facing up are odd. All of the yellow cards facing up are even.

Next, in the second round of flips, E Z flips over every card that has a number which is a multiple of 3. Let us look at how many red cards will be flipped over to yellow and how many yellow cards will be flipped over to red.

There are 33 multiples of three from 1 to 100,

$$\{3, 6, 9, 12, 15, \dots, 87, 90, 93, 96, 99\}.$$

Of these numbers, 17 are odd and 16 are even. But the 17 odd multiples of 3 are currently red and therefore are flipped over to become yellow. The 16 even multiples of 3 are currently yellow and are therefore flipped over to become red (again).

So after the first flip there were 50 red and 50 yellow. Of the 50 red, 17 were flipped to yellow. Of the 50 yellow, 16 of them were flipped to red. Therefore after the second round, there were $50 - 17 + 16 = 49$ red cards.

For Further Thought

Suppose the game were to be played as follows: in each round you flip all of the multiples of the prime numbers in order from 2 to 97. The first two rounds of the new version of the game were played above. The next round would be multiples of 5, and then multiples of 7, and so on until the final round is reached.

After going through all of the rounds, how many cards have the red side facing up?

