Problem of the Week
Problem D and Solution
Three by Three

Problem

The first 9 positive odd integers are placed in the 3×3 grid shown to the right in such a way that the sum of each row, column and diagonal is the same. Four of the numbers are shown and the other five numbers are hidden behind the letters A, B, C, D, and E. Determine the value of $A + E$.

A	1	B
5	C	13
D	E	3

Solution

Solution 1

The numbers to be placed in the table are 1, 3, 5, 7, 9, 11, 13, 15, and 17, the first 9 positive odd integers. The sum of all the numbers in the table is $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 = 81$. It follows that the sum of the sums of the three rows is 81. But each row has the same sum so the sum of each row is $81 \div 3 = 27$. We know that each row, column and diagonal has the same sum. Therefore the sum of each row = the sum of each column = the sum of each diagonal = 27.

We can now use this information to determine the values in each cell of the 3×3 grid. In the third column we know that $B + 13 + 3 = 27$ and $B = 11$ follows.

In the first row we know that $A + 1 + B = 27$ and $B = 11$ so $A + 1 + 11 = 27$. $A = 15$ follows.

In the second row we know that $5 + C + 13 = 27$ and $C = 9$ follows.

Then in the second column $1 + C + E = 27$ and $C = 9$ so $1 + 9 + E = 27$. $E = 17$ follows.

Since we know the values of A and E we can compute the sum $A + E = 15 + 17 = 32$.

Therefore, the sum $A + E$ is 32.

Solution 2

In the second solution we determine the required sum without finding the row/column/diagonal sum. Since the row sum equals the column sum we know that the sum of row 1 equals the sum of column 3.

$$\begin{aligned} A+1+B &= B+13+3 \\ A+1 &= 13+3 && \text{since } B \text{ is common to both sides} \\ \therefore A &= 15 \end{aligned}$$

Again, since the row sum equals the column sum we know that the sum of column 2 equals the sum of row 2.

$$\begin{aligned} 1+C+E &= 5+C+13 \\ 1+E &= 5+13 && \text{since } C \text{ is common to both sides} \\ \therefore E &= 17 \end{aligned}$$

Since we know the values of A and E we can compute the sum $A + E = 15 + 17 = 32$.

Therefore, the sum $A + E$ is 32.

Solution 3

In solution 3 we find the sum $A + E$ without finding any individual values.

$$\begin{aligned} \text{Sum of Row 1} + \text{Sum of Column 2} &= \text{Sum of Row 2} + \text{Sum of Column 3} \\ A+1+B+1+C+E &= 5+C+13+B+13+3 \end{aligned}$$

Since $B + C$ is common to both sides, the equation simplifies to:

$$\begin{aligned} A+1+1+E &= 5+13+13+3 \\ A+E+2 &= 34 \\ \therefore A+E &= 32 \end{aligned}$$

Therefore, the sum $A + E$ is 32.