



## Problem of the Week

### Problem D and Solution

### Who You Gonna Call?

#### Problem

Some businesses, government offices, schools, and hospitals have a phone fan out system that is used to quickly distribute important information to all of its employees. A certain school, with a total of 100 employees, wishes to create a phone fan out system to be used in the event of an emergency school closure. The principal contacts three other employees, each of whom contacts three others, and so on, until all of the employees of the school have been contacted.

Determine the maximum number of people who do not need to make a call in this system.

#### Solution

##### Solution 1

It is important to note at the outset that in order to minimize the number of callers, we need to maximize the number of calls made by those who do make calls.

Once the principal makes the initial phone call, four people (the principal and three others) have the information. There are  $100 - 4 = 96$  others to contact.

The next three people make three calls each or a total of 9 calls. Now 13 people have the information and 87 people still need to be contacted.

The next 9 people make 3 calls each or a total of 27 calls. Now 40 people have the information and 60 people still need to be contacted.

In order to reach the final 60 people, only  $60 \div 3 = 20$  more people need to make calls. The total number of people required to make calls is  $1 + 3 + 9 + 20 = 33$ .

Therefore,  $100 - 33 = 67$  is the maximum number of people who do not need to make calls.

A system like this is actually still very efficient at getting information to a large number of people, each of whom makes a small number of calls. Close to one third of the people in the organization need to call. About two-thirds of the people in the organization do not need to make calls.





## Solution 2

It is important to note at the outset that in order to minimize the number of callers, we need to maximize the number of calls made by those who do make calls.

This solution is similar to the first solution at the beginning. The change comes after the group of 27 have been called.

Once the principal makes the initial phone call, four people (the principal and three others) have the information. There are  $100 - 4 = 96$  others to contact.

The next three people make three calls each or a total of 9 calls. Now 13 people have the information and 87 people still need to be contacted.

The next 9 people make 3 calls each or a total of 27 calls. Now 40 people have the information and 60 people still need to be contacted.

If the next 27 people make 3 calls each or a total of 81 calls, this is  $81 - 60 = 21$  calls too many.

There are  $21 \div 3 = 7$  of the 27 people that do not need to make a call. Thus, only  $27 - 7 = 20$  people need to make a call. This gives a total of  $1 + 3 + 9 + 20 = 33$  people required to make calls and  $100 - 33 = 67$  people who do not need to make calls.

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