



# Problem of the Week

## Problem D and Solution

### Top Secret



#### Problem

Every Friday, five spies, Spy A, Spy B, Spy C, Spy D, and Spy E, share all of the information they have uncovered over the previous week. A spy can never be seen with more than one other spy at the same time. They must conduct several rounds of meetings where two pairs of spies meet at different locations in order to share all of the information they have to that point. In any round, there are always 2 meetings involving 4 of the spies. During a round one spy is not involved in any meeting. There are two methods to conduct the rounds of meetings.

In the first method, at some point on Friday each spy meets with every other spy directly. In their meeting, a spy shares only the information they gathered themselves during the previous week. They do not share information learned from other spies in previous meetings.

In the second method, when two spies meet each spy is able to reliably pass along all of the information learned in previous meetings plus their own information which they gathered in the previous week. That is, in any meeting each spy gets all of the information known to the other spy.

For each method, determine the minimum number of rounds of meetings required in order for each spy to learn all of the information gathered by each of the other spies during the previous week.

#### Solution

For the **first method** we must determine the number of meetings required so that each spy can meet face-to-face with each of the other spies. Then we will determine the number of rounds needed in order to schedule all of the meetings.

Spy A will meet face to face with each of the other four spies. This requires 4 meetings: Spy A with Spy B, Spy A with Spy C, Spy A with Spy D, and Spy A with Spy E.

Spy B must meet face to face with each of the other four spies. However, Spy B has already met with Spy A. So Spy B requires 3 more meetings: Spy B with Spy C, Spy B with Spy D, and Spy B with Spy E. There will now be a total of 7 meetings.

Spy C must meet face to face with each of the other four spies. However, Spy C has already met with Spy A and Spy B. So Spy C requires 2 more meetings: Spy C with Spy D, and Spy C with Spy E. There will now be a total of 9 meetings.

Spy D must meet face to face with each of the other four spies. However, Spy D has already met with Spy A, Spy B and Spy C. So Spy D only requires 1 more meeting: Spy D with Spy E. There will now be a total of 10 meetings.

At this point Spy E will have met face to face with each of the other four spies so no further meetings are needed. A total of 10 meetings are required.





The spies require a total of 10 meetings but how many rounds will this take? During any round only 4 of the spies can be in meetings. Can we arrange the 10 meetings to fit in 5 rounds with two meetings per round? With a little “playing”, this can easily be done in many ways. One such way is as follows:

Round Number	Meeting 1	Meeting 2
1	Spy A meets with Spy B	Spy C meets with Spy D
2	Spy A meets with Spy C	Spy B meets with Spy E
3	Spy A meets with Spy D	Spy C meets with Spy E
4	Spy A meets with Spy E	Spy B meets with Spy D
5	Spy B meets with Spy C	Spy D meets with Spy E

There should be 5 rounds of meetings so that each spy meets with each of the other spies directly to share their own information face-to-face.

The **second method** assumes that once information is shared between two spies, each spy knows all of the information that was shared in the meeting.

In the first round, two meetings can take place. Suppose Spy A meets with Spy B, and Spy C meets with Spy D. At this point, Spy A and Spy B each know their own information as well as the information gathered by the other spy. Also, at this point, Spy C and Spy D know their own information and the information gathered by the other spy. This means, however, that Spy E was not involved and Spy E’s information is only known by Spy E. Therefore, it is not possible to know all of the information after one round.

In the second round, Spy E can meet with Spy A or Spy B and gain the information common to each of them or meet with Spy C or Spy D and gain the information common to each of them. Suppose Spy E meets with Spy A. Now Spy A knows the information from Spy B and Spy E but not Spy C or Spy D. Spy B can meet with Spy C or Spy D and gain the information common to each of them. Suppose Spy B meets with Spy C. Now Spy B knows the information from Spy A, Spy C and Spy D but not Spy E. In this round, Spy D sat out.

After two rounds,

Spy	Knows Information From Spies
A	A, B, E
B	A, B, C, D
C	A, B, C, D
D	C, D
E	A, B, E





No matter how the meetings are arranged to the end of the second round, no spy can know all of the information. The most that can be known by any one spy is their own information plus the information gathered from three of the other spies. It follows that it is not possible for every spy to know all of the information gathered by each of the other spies after 2 rounds.

Can all of the information be shared by the end of round 3? Since there is an odd number of spies, one of the spies must sit out each round. None of the spies have all of the information before the start of round 3. So whichever spy sits out of round 3 will not have all of the information. It is not possible to share all of the information in three rounds.

However, if in round 3 Spy A meets with Spy C and Spy D meets with Spy E, then at the end of round 3 the information situation is:

Spy	Knows Information From Spies
A	A, B, C, D, E
B	A, B, C, D
C	A, B, C, D, E
D	A, B, C, D, E
E	A, B, C, D, E

In round 4, Spy B can meet with any other spy to learn the information known by each of the other spies. So that observers do not become suspicious, perhaps Spy B should meet with a different spy, say Spy E. No other meeting need take place in this round.

The minimum number of rounds required for every spy to learn the information known by each spy is 4.

For Further Thought: Suppose there were 6 spies. Using the second method determine the minimum number of rounds of meetings required so that all of the information known by each spy has been shared with every other spy. You may be surprised by the result.

