



# Problem of the Week

## Problem D and Solution

### An Age-Old Problem

#### Problem

At present, the sum of the ages of a father and his son is 43 years. In  $n$  years,  $n > 0$ , the father will be four times the son's age.

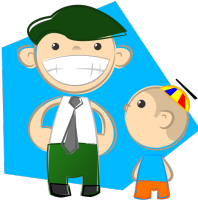
Determine the possible ages of the father and son.

#### Solution

Let  $s$  represent the present age of the son and  $f$  represent the present age of the father.

Since the sum of their present ages is 43,  $s + f = 43$  or  $f = 43 - s$ .

In  $n$  years, the son will be  $(s + n)$  years old and the father will be  $(43 - s + n)$  years old. At that time the father will be four times older than the son.



$$\begin{aligned} \therefore 43 - s + n &= 4(s + n) \\ 43 - s + n &= 4s + 4n \\ 43 &= 5s + 3n \\ \frac{43 - 3n}{5} &= s \end{aligned}$$

We are looking for integer values of  $n$  so that  $43 - 3n$  is divisible by 5.

When  $n = 1$ ,  $s = \frac{43-3n}{5} = \frac{43-3(1)}{5} = \frac{40}{5} = 8$ . When  $s = 8$ ,  $f = 43 - s = 43 - 8 = 35$ .

When  $n = 6$ ,  $s = \frac{43-3n}{5} = \frac{43-3(6)}{5} = \frac{25}{5} = 5$ . When  $s = 5$ ,  $f = 43 - s = 43 - 5 = 38$ .

When  $n = 11$ ,  $s = \frac{43-3n}{5} = \frac{43-3(11)}{5} = \frac{10}{5} = 2$ . When  $s = 2$ ,  $f = 43 - s = 43 - 2 = 41$ .

When  $n = 16$ ,  $s = \frac{43-3n}{5} = \frac{43-3(16)}{5} = \frac{-5}{5} = -1$ . Since  $s < 0$ ,  $n = 16$  does not produce a valid age for the son. No higher value of  $n$  would produce a value of  $s > 0$ .

No integer values of  $n$  other than 1, 6, and 11, between 0 and 16, produce a multiple of 5 when substituted into  $43 - 3n$ .

If today the father is 35 and the son is 8, then in 1 year the father will be 36 and the son will be 9. The father will be four times older than the son since  $4 \times 9 = 36$ .

If today the father is 38 and the son is 5, then in 6 years the father will be 44 and the son will be 11. The father will be four times older than the son since  $4 \times 11 = 44$ .

If today the father is 41 and the son is 2, then in 11 years the father will be 52 and the son will be 13. The father will be four times older than the son since  $4 \times 13 = 52$ .

Therefore the possible present ages for the father and son are 35 and 8 or 38 and 5 or 41 and 2.

