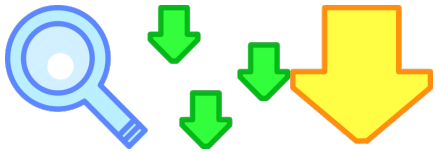


Problem of the Week
Problem D and Solution
Download, Download, ...

Problem

Salim wants to download a certain number of his favourite tunes using some money he has saved doing odd jobs. At Downloads For U, they charge \$1.00 per download. He would need to save \$17.00 more than he already has in order to download all he wants. At a competing site, Music Bytes, they only charge \$0.80 per download. He would have \$5.00 left over from his savings if he downloads all he wants. How many downloads does Salim want?

Solution

Solution 1

Let n represent the number of songs that Salim wants to download.

Since each song at Downloads For U costs \$1.00, Salim would spend $1 \times n = n$ dollars in total. If Salim were to purchase all of the songs he wants at Downloads For U, he would be short \$17 dollars in his savings. Therefore, the amount he has in his savings is $(n - 17)$ dollars.

Since each song at Music Bytes costs \$0.80, Salim would spend $0.8 \times n = 0.8n$ dollars in total. If Salim were to purchase all of the songs he wants at Music Bytes, he would have \$5 dollars left over in his savings. Therefore, the amount he has in his savings is $(0.8n + 5)$ dollars.

We have two expressions for the amount in his savings so we can establish the equality $n - 17 = 0.8n + 5$. This simplifies to $0.2n = 22$. After dividing each side by 0.2, we obtain $n = 110$.

Therefore, Salim wants to purchase a total of 110 downloads.

Solution 2

Let n represent the number of songs that Salim wants to download.
Let x represent the amount that Salim has in his savings.

Since the difference between the costs of a single download is \$1.00 − \$0.80 = \$0.20, the total cost difference of purchasing n downloads would be $\$0.2n$.

To purchase from Downloads For U, Salim would need to spend \$17 more than he has in his savings. Therefore, he would need $(x + 17)$ dollars. To purchase from Music Bytes, Salim would need to spend \$5 less than he has in his savings. Therefore, he would need $(x - 5)$ dollars. The total cost difference of purchasing n downloads would be $(x + 17) - (x - 5) = 22$ dollars.

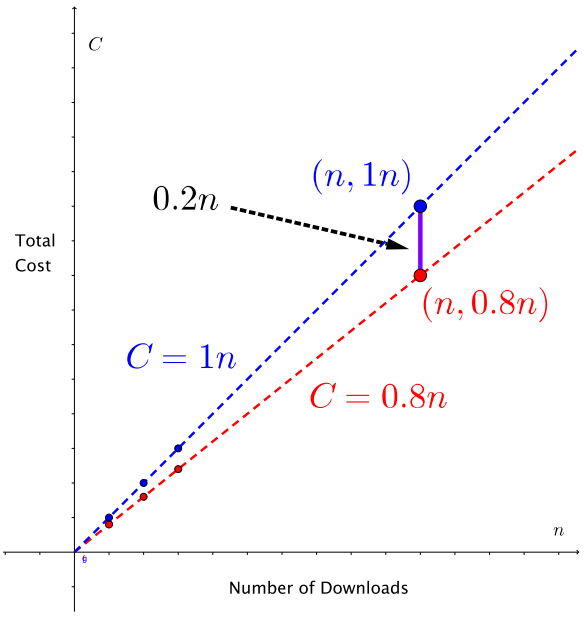
We have two expressions for the cost difference and can establish the following equality $0.2n = 22$. After dividing each side by 0.2, we obtain $n = 110$.

Therefore, Salim wants to purchase a total of 110 downloads.

We could now easily determine the value of his savings but will leave this for the reader.

Solution 3

This solution looks at the problem graphically.



Let n represent the number of downloads purchased where n is an non-negative integer.
Let C represent the cost of purchasing n downloads.

At Downloads For U, each download costs \$1 so $C = 1n$. This relation is a set of distinct points with non-negative integer x-coordinates that lie on the line $C = n$. On the graph above, this relation is the set of points which lie on the upper dotted line. A general point on the line is $(n, 1n)$.

At Music Bytes, each download costs \$0.80 so $C = 0.8n$. This relation is a set of distinct points with non-negative integer x-coordinates that lie on the line $C = 0.8n$. On the graph above, this relation is the set of points on the lower dotted line. A general point on the line is $(n, 0.8n)$.

The length of any vertical line segment connecting points on the two lines would represent the cost difference between the two stores when a given number of downloads are purchased. This length is the difference between y-coordinates. If n downloads are purchased, the difference in total cost is $1n - 0.8n = 0.2n$.

Since the amount of Salim's savings is constant, we can calculate the total cost difference. For the first plan he would need \$17 more than he has. For the second plan he would need \$5 less than what he has. The total cost difference would be $17 - (-5) = 22$ dollars.

We have two expressions for the total cost difference and can establish the following equality $0.2n = 22$. After dividing each side by 0.2, we obtain $n = 110$.

Therefore, Salim wants to purchase a total of 110 downloads.

We could now easily determine the value of his savings but will leave this for the reader.