Problem of the Week
Problem D and Solution
Unknown Volume

Problem
The side, front and top faces of a rectangular solid have areas of $\frac{x}{3}, 3y,$ and $100xy$ cm$^2$, respectively. Determine the volume of the rectangular solid in terms of $x$ and $y$.

Solution
Since $3y$ and $\frac{x}{3}$ are surface areas, then $x$ and $y$ must be positive. Let the length, height and width of the rectangular solid be $a$, $b$, $c$, respectively.

To determine the volume we need to find the product $abc$.

Since the area of the side is $\frac{x}{3}$, then $bc = \frac{x}{3}$. (1)
Since the area of the front is $3y$, then $ab = 3y$. (2)
Since the area of the top is $100xy$, then $ac = 100xy$. (3)

Multiplying the left sides and multiplying the right sides of each of the equations (1), (2) and (3), we obtain:

\[(bc)(ab)(ac) = \left(\frac{x}{3}\right)(3y)(100xy)\]
\[a^2b^2c^2 = 100x^2y^2\]
\[(abc)^2 = (10xy)^2\]

Taking the square root, $abc = 10xy$, since all quantities are positive.

But $abc$ is the volume. Therefore the volume of the rectangular solid, in terms of $x$ and $y$, is $10xy$ cm$^3$. 