Problem of the Week
Problem D and Solution
The Distance Across

Problem
Brett wishes to design a tile as shown. He will start with a white square $ABCD$ and then colour two pairs of identical isosceles triangles leaving the white rectangle $KLMN$. The design has a line from $M$ to $K$. If the total area of the coloured sections is $50 \text{ cm}^2$, what is the length of $MK$?

Solution
Let $x$ represent the lengths of the equal sides of the larger triangles and $y$ represent the lengths of the equal sides of the smaller triangles.

Now, area $\triangle AMN = \text{area } \triangle LKC = \frac{1}{2}x^2$.

Also, area $\triangle MDL = \text{area } \triangle BNK = \frac{1}{2}y^2$.

Therefore the total area of the triangles is $\frac{1}{2}x^2 + \frac{1}{2}x^2 + \frac{1}{2}y^2 + \frac{1}{2}y^2 = x^2 + y^2 = 50$.

Using Pythagoras’ Theorem and since $MK$ is a diagonal of rectangle $MNKL$, then $MK^2 = MN^2 + NK^2$.

Now to find the length of $MK$, two different solutions are provided.
Solution 1

In \( \triangle AMN \), \( MN^2 = AM^2 + AN^2 = x^2 + x^2 \)
and in \( \triangle BNK \), \( NK^2 = BN^2 + BK^2 = y^2 + y^2 \).
Now substituting into \( MK^2 = MN^2 + NK^2 \),
\[
MK^2 = MN^2 + NK^2 = x^2 + x^2 + y^2 + y^2 = 50 + 50 = 100
\]
\( MK = 10, MK > 0 \).
Therefore the length of \( MK \) is 10 cm.

Solution 2

In \( \triangle AMN \), \( MN^2 = x^2 + x^2 = 2x^2 \). Therefore, \( MN = \sqrt{2}x, x > 0 \).
In \( \triangle BNK \), \( NK^2 = y^2 + y^2 = 2y^2 \). Therefore, \( NK = \sqrt{2}y, x > 0 \).
Now substituting into \( MK^2 = MN^2 + NK^2 \),
\[
MK^2 = MN^2 + NK^2 = 2x^2 + 2y^2 = 2(50) = 100
\]
\( MK = 10, MK > 0 \)
Therefore, the length of \( MK \) is 10 cm.