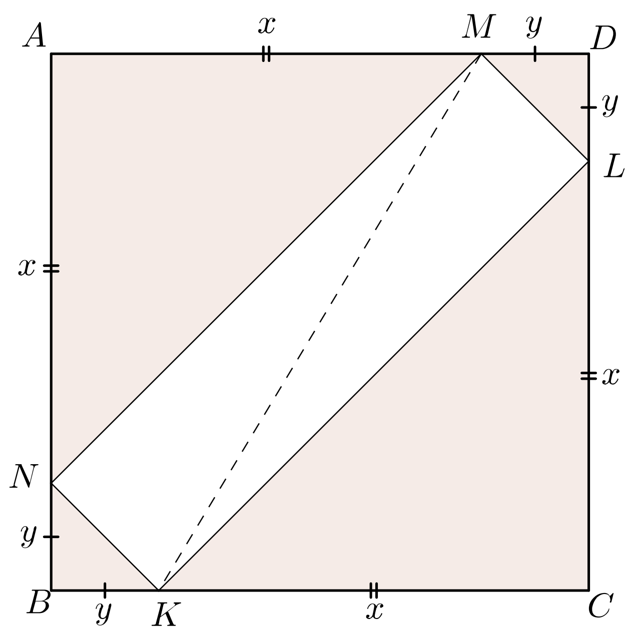
Problem of the Week

Problem D and Solution

The Distance Across

Problem

Brett wishes to design a tile as shown. He will start with a white square $ABCD$ and then colour two pairs of identical isosceles triangles leaving the white rectangle $KLMN$. The design has a line from M to K. If the total area of the coloured sections is 50 cm^2, what is the length of MK?



Solution

Let x represent the lengths of the equal sides of the larger triangles and y represent the lengths of the equal sides of the smaller triangles.

Now, area $\triangle AMN$ = area $\triangle LKC = \frac{1}{2}x^2$.
Also, area $\triangle MDL$ = area $\triangle BNK = \frac{1}{2}y^2$.

Therefore the total area of the triangles is $\frac{1}{2}x^2 + \frac{1}{2}x^2 + \frac{1}{2}y^2 + \frac{1}{2}y^2 = x^2 + y^2 = 50$.

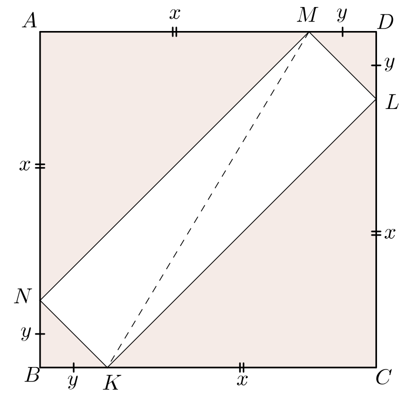
Using Pythagoras' Theorem and since MK is a diagonal of rectangle $MNKL$,

$$\text{then } MK^2 = MN^2 + NK^2.$$

Now to find the length of MK, two different solutions are provided.

Solution 1

In $\triangle AMN$, $MN^2 = AM^2 + AN^2 = x^2 + x^2$
and in $\triangle BNK$, $NK^2 = BN^2 + BK^2 = y^2 + y^2$.
Now substituting into $MK^2 = MN^2 + NK^2$,



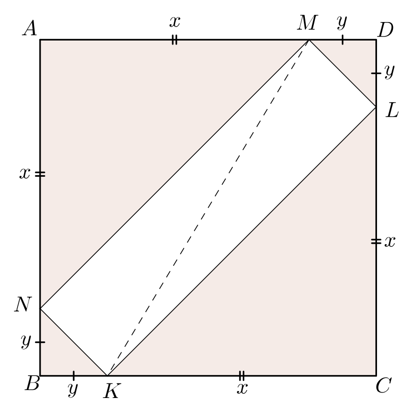
$$\begin{aligned} MK^2 &= MN^2 + NK^2 \\ &= x^2 + x^2 + y^2 + y^2 \\ &= x^2 + y^2 + x^2 + y^2 \\ &= 50 + 50 \\ &= 100 \\ MK &= 10, MK > 0. \end{aligned}$$

Therefore the length of MK is 10 cm.

Solution 2

In $\triangle AMN$, $MN^2 = x^2 + x^2 = 2x^2$. Therefore, $MN = \sqrt{2}x, x > 0$.
In $\triangle BNK$, $NK^2 = y^2 + y^2 = 2y^2$. Therefore, $NK = \sqrt{2}y, x > 0$.

Now substituting into $MK^2 = MN^2 + NK^2$,



$$\begin{aligned} MK^2 &= MN^2 + NK^2 \\ &= (\sqrt{2}x)^2 + (\sqrt{2}y)^2 \\ &= 2x^2 + 2y^2 \\ &= 2(x^2 + y^2) \\ &= 2(50) \\ &= 100 \\ MK &= 10, MK > 0 \end{aligned}$$

Therefore, the length of MK is 10 cm.