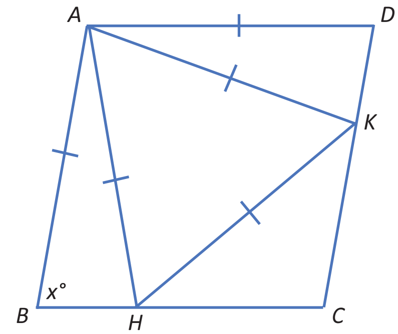
Problem of the Week

Problem D and Solution

Roaming in a Rhombus

Problem

A *parallelogram* is a quadrilateral with opposite sides parallel. A *rhombus* is a parallelogram with all four sides having equal length. $ABCD$ is a rhombus. H is on BC, between B and C, and K is on CD, between C and D, such that $AB = AH = HK = KA$. Determine the measure of $\angle BAD$.



Solution

Draw the diagram based on the given information. Since $ABCD$ is a rhombus, we know $AB = BC = CD = DA$. Let $\angle ABH = x^\circ$.

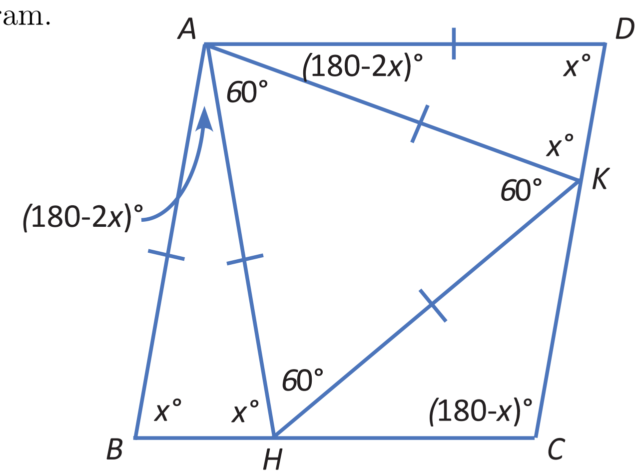
Since $AH = HK = KA$, $\triangle AHK$ is equilateral and each angle is 60°. In particular, $\angle HAK = 60^\circ$.

In $\triangle ABH$, $AB = AH$ and the triangle is isosceles. Therefore, $\angle AHB = \angle ABH = x^\circ$. Then $\angle BAH = (180 - 2x)^\circ$.

Since $ABCD$ is a rhombus, $AB \parallel CD$ and $\angle ABC + \angle BCD = 180^\circ$. It follows that $\angle BCD = (180 - x)^\circ$. But in the rhombus $BC \parallel AD$ and $\angle BCD + \angle ADC = 180^\circ$. It follows that $\angle ADC = 180^\circ - (180 - x)^\circ = x^\circ$.

In $\triangle AKD$, $KA = AD$ and the triangle is isosceles. Therefore, $\angle AKD = \angle ADK = x^\circ$. Then $\angle DAK = (180 - 2x)^\circ$.

All of this new information is shown on the second diagram.



Since $ABCD$ is a rhombus, $BC \parallel AD$ and

$$\begin{aligned}
\angle BAD &= 180^\circ - \angle ABC \\
(180 - 2x)^\circ + 60^\circ + (180 - 2x)^\circ &= (180 - x)^\circ \\
(420 - 4x)^\circ &= (180 - x)^\circ \\
240^\circ &= (3x)^\circ \\
\therefore x^\circ &= 80^\circ \\
(180 - x)^\circ &= 100^\circ
\end{aligned}$$

But $\angle BAD = (180 - x)^\circ$.

$\therefore \angle BAD = 100^\circ$.