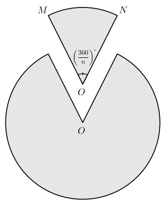

Problem of the Week

Problem D and Solution

It Matters How You Slice It

Problem

MON is a sector of a circle with centre O, radius n and $\angle MON = \left(\frac{360}{n}\right)^\circ$. Determine all positive integers n for which the sector MON has perimeter greater than 20 and less than 30.

Solution

As the sector angle gets larger, so does the length of the arc if the radius remains the same. In this problem, as the radius n increases, the sector angle $\left(\frac{360}{n}\right)^\circ$ decreases. It is difficult to "see" what happens to the length of the arc.

Since the ratio of the arc length to the circumference of the circle is the same as the ratio of the sector angle to 360°,

$$\begin{aligned}\text{Arc Length} &= \frac{\text{Sector Angle}}{360^\circ} \times \text{circumference} \\ &= \frac{\frac{360}{n}}{360} \times \pi d \\ &= \frac{1}{n} \times \pi \times 2n, \qquad \text{since } d = 2r = 2n \\ &= 2\pi\end{aligned}$$

As the radius increases, the sector angle decreases and the arc length MN remains constant, 2π units.

$$\begin{aligned}\text{Perimeter} &= MO + ON + \text{ arc length } MN \\ &= n + n + 2\pi \\ &= 2n + 2\pi\end{aligned}$$

We want all integer values of n such that:

$$\begin{aligned}20 < 2n + 2\pi \quad &\text{and} \quad 2n + 2\pi < 30 \\ 10 < n + \pi \quad &\text{and} \quad n + \pi < 15 \\ 10 - \pi < n \quad &\text{and} \quad n < 15 - \pi\end{aligned}$$

We want all integer values of n such that $n > 10 - \pi \doteq 6.9$ and $n < 15 - \pi \doteq 11.9$. The only integer values of n that satisfy these conditions are $n = 7$, $n = 8$, $n = 9$, $n = 10$, and $n = 11$.