



## Problem of the Week

### Problem D and Solution

#### True or False



#### Problem

On an island, there are two types of inhabitants: Heroes, who always tell the truth, and Villains, who always lie. Four of the inhabitants of the island are seated around a circular table. When asked, “Are you a Hero or a Villain?”, all four replied, “Hero.” When asked, “Is the person on your right a Hero or a Villain?”, all four replied, “Villain.” How many Heroes are there seated at the table? Verify that your solution is the only possible solution.

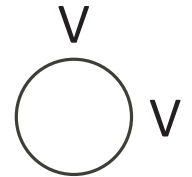
#### Solution

There are really five possibilities to check: there could be four Heroes, there could be four Villains, there could be three Villains and one Hero, there could be three Heroes and one Villain, or there could be two Villains and two Heroes.

We can eliminate cases as follows:

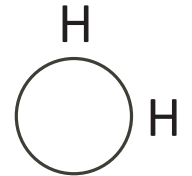
##### 1. Can two Villains ever sit beside each other?

Since Villains always lie, when the two Villains sitting beside each other answer the first question they will both lie and say “Hero”. However, in responding to the second question, the Villain with the other Villain on the right hand side would have to lie and say “Hero”. But everyone responded “Villain”. This is a contradiction and there can therefore never be two Villains seated beside each other. This conclusion effectively eliminates the possibility that there are four Villains or three Villains and one Hero.



##### 2. Can two Heroes ever sit beside each other?

Since Heroes always tell the truth, when the two Heroes sitting beside each other answer the first question they will both tell the truth and say “Hero”. However, in responding to the second question, the Hero with the other Hero on the right hand side would have to tell the truth and say “Hero”. But everyone responded “Villain”. This is a contradiction and therefore there can never be two Heroes seated beside each other. This conclusion effectively eliminates the possibility that there are four Heroes or three Heroes and one Villain.



The only possibility left is that there are two Villains and two Heroes such that no two Villains or two Heroes can sit next to each other. The diagram illustrates how they must sit relative to each other. We can confirm that this arrangement satisfies the problem. Since all Villains lie and all Heroes tell the truth, they must all answer the first question “Hero”. Since all Villains lie and all Heroes tell the truth, they must all answer the second question “Villain”. Therefore, there are two Heroes and two Villains, and when seated at a circular table they alternate Hero, Villain, Hero, Villain.

