



## Problem of the Week

### Problem C and Solution

#### You Can't Go Back?



#### Problem

On your 13th birthday you received three different time-travel pedometers. You want to use your pedometers to travel back to your 8th birthday. You may use your pedometers as often as you wish but only one at a time. Each time you use Pedometer A, take exactly 7 steps forward. This will result in you going back 4 months in time. Each time you use Pedometer B, take exactly 5 steps backward. This will result in you going back 7 months in time. Each time you use Pedometer C, take exactly 2 steps forward. This will result in you going back 3 months in time.

In traveling back to your 8th birthday, you made a total of 25 backward steps and had a total of 12 pedometer uses. How many forward steps did you take while using your pedometers?

#### Solution

From your 13th birthday to your 8th birthday you would travel 5 years back in time. This is equivalent to traveling  $5 \times 12 = 60$  months back in time.

Pedometer B is the only pedometer that requires its user to step backward. For every 5 steps backward, you travel 7 months back in time. Therefore, for 25 steps backward, you use Pedometer B five times and travel back in time  $5 \times 7 = 35$  months.

You still need to travel  $60 - 35 = 25$  more months back in time. You have used a pedometer 5 times and since you only have a total of 12 pedometer uses, you have  $12 - 5 = 7$  pedometer uses left. You can now only use Pedometer A and Pedometer C.

If you use Pedometer A and Pedometer C one time each, you travel a total of 7 months back in time. If you use Pedometer A and Pedometer C three times each, this accounts for 6 uses and you travel a total of  $7 \times 3 = 21$  months back in time. You have 1 use left and still need to travel 4 more months back in time. This can be accomplished by using Pedometer A once more.

It follows that Pedometer A is used 4 times and Pedometer C is used 3 times. The total number of forward steps is  $4 \times 7 + 3 \times 2 = 28 + 6 = 34$ .

Note that we could also have looked at each of the possibilities for using Pedometer A. Since there are a total of 7 pedometer uses for Pedometers A and C, the minimum number of uses for Pedometer A would be 0 and the maximum number of uses for Pedometer A would be 7. Once the number of uses for Pedometer A is selected, the number of uses for Pedometer C could be determined by subtracting the number of uses for Pedometer A from 7. For each combination we could determine the number of months traveled back in time. Once the correct combination is determined the total number of forward steps can be calculated. This is summarized in a table on the next page.

An algebraic solution is also provided on the next page.





Using only Pedometer A and Pedometer C a total of 7 times, we want to travel back in time 25 months.

Uses of Pedometer A	Uses of Pedometer C	Months Traveled Back in Time
0	7	$0 \times 4 + 7 \times 3 = 0 + 21 = 21$
1	6	$1 \times 4 + 6 \times 3 = 4 + 18 = 22$
2	5	$2 \times 4 + 5 \times 3 = 8 + 15 = 23$
3	4	$3 \times 4 + 4 \times 3 = 12 + 12 = 24$
4	3	$4 \times 4 + 3 \times 3 = 16 + 9 = 25$
5	2	$5 \times 4 + 2 \times 3 = 20 + 6 = 26$
6	1	$6 \times 4 + 1 \times 3 = 24 + 3 = 27$
7	0	$7 \times 4 + 0 \times 3 = 28 + 0 = 28$

Only one combination gives the correct number of Pedometer uses and the correct number of months traveled back in time. Use Pedometer A 4 times and Pedometer C 3 times. The total number of forward steps is  $4 \times 7 + 3 \times 2 = 28 + 6 = 34$ .

### Algebraic Approach

This solution is presented for you to get a glimpse of what is coming in future mathematics courses.

Let  $a$  be the number of uses of Pedometer A,  $b$  be the number of uses of Pedometer B, and  $c$  be the number of uses of Pedometer C. Since the total number of uses is 12, then  $a + b + c = 12$ .

The total number of backward steps is 25 and Pedometer B is the only pedometer requiring backward steps. Since each use of Pedometer B requires 5 backward steps, then we require a total of 5 uses of Pedometer B to go back 25 steps. It follows that  $b = 5$  and the equation  $a + b + c = 12$  becomes  $a + 5 + c = 12$  which simplifies to  $a + c = 7$ . (1)

In using Pedometer B 5 times, you travel a total of  $5 \times 7 = 35$  months back in time. You need to travel 5 years or 60 months back in time altogether. Using Pedometer A and Pedometer C a total of 7 times, you need to travel  $60 - 35 = 25$  more months back in time. As an equation this can be written  $4a + 3c = 25$ . (2)

Rearranging equation (1), we obtain  $c = 7 - a$ . We can substitute for  $c$  in equation (2).

$$\begin{aligned} 4a + 3c &= 25 \\ 4a + 3(7 - a) &= 25 \\ 4a + 21 - 3a &= 25 \\ a + 21 &= 25 \\ a &= 4 \end{aligned}$$

Since  $a = 4$ , we can substitute in equation (1) to determine that  $c = 3$ .

For each use of Pedometer A, 7 forward steps are required. Therefore, you step forward  $7a$  steps using Pedometer A. For each use of Pedometer C, 2 forward steps are required.

Therefore, you step forward  $2c$  steps using Pedometer C. The total number of steps forward is  $7a + 2c$ . But  $a = 4$  and  $c = 3$  so the total number of forward steps is  $7(4) + 3(2) = 28 + 6 = 34$ .

