Problem of the Week
Problem C and Solution
How tall are you?

Problem

There are 9 students divided into two groups, the Nanos and the Technos. The heights (in cm) of the 9 students are 151, 153, 157, 161, 153, 157, 156, 159, and 154. The following information is known about the two groups. No Nano is taller than any Techno but one of the Nanos is the same height as one of the Technos. Two of the Nanos are the same height.

Determine the difference between the mean height of a member of the Technos and the mean height of a member of the Nanos.

Solution

Place the heights in ascending order: 151 153 153 154 156 157 157 159 161.

Since no Nano is taller than any Techno we can introduce a separator "|" that will divide the Nanos and Technos into two distinct groups.

For example, if there are 4 Nanos and 5 Technos, we would place a separator between 154 and 156.

(Nanos in front) 151 153 153 154 | 156 157 157 159 161 (Technos at the end)

Now, the possibilities for the two Nanos with the same height are 157 cm or 153 cm.

If the Nanos with the same height are both 157 cm, then the Nano and Techno with the same height would both be 153 cm. This contradicts the fact that no Nano is taller than any Techno because there would be a Nano whose height is 157 cm and a shorter Techno whose height is 153 cm.

Therefore, the two Nanos with the same height must both be 153 cm, and the Nano and the Techno with the same height must both be 157 cm.

Nanos	Technos
151 153 153 154 156 157	157 159 161

The mean height of a student in the Nano group is $\frac{151+153+153+154+156+157}{6} = \frac{924}{6} = 154$ cm.

The mean height of a student in the Techno group is $\frac{157+159+161}{3} = \frac{477}{3} = 159$ cm.

Therefore, the difference in mean heights between the students in the Techno group and the students in the Nano group is $59 - 54 = 5$ cm.

NOTE: This question is similar to a question found on the Beaver Computing Challenge (BCC) which is usually written in November. BCC information and past challenges can be found at http://cemc.uwaterloo.ca/contests/bcc.html .