



## Problem of the Week

### Problem C and Solution

#### Moving Right Along

#### Problem

Six coloured squares are placed beside each other as shown below.



The leftmost square has the number 504 on it and the rightmost square has the number 2017 on it. A number is to be written on each of the four blank squares so that each number after the second number equals the sum of the previous two numbers.

Determine the remaining four numbers that should be written on the front of the boxes as you move from left to right.

#### Solution

Let  $a$  represent the second number.

Since the third number is the sum of the previous two numbers, the third number is  $504 + a$ .

Since the fourth number is the sum of the previous two numbers, the fourth number is  $(a) + (504 + a) = 504 + 2a$ .

Since the fifth number is the sum of the previous two numbers, the fifth number is  $(504 + a) + (504 + 2a) = 1008 + 3a$ .

Since the sixth number is the sum of the previous two numbers, the sixth number is  $(504 + 2a) + (1008 + 3a) = 1512 + 5a$ . But the sixth number in the sequence is 2017.

$$\begin{aligned}\therefore 1512 + 5a &= 2017 \\ 1512 + 5a - 1512 &= 2017 - 1512 \\ 5a &= 505 \\ \frac{5a}{5} &= \frac{505}{5} \\ a &= 101\end{aligned}$$

We now know that the second number is 101 so we can determine the remaining numbers in the sequence by substituting into the expressions above or by simply using the rule to generate the remaining numbers. Using the rule, the third number is  $504 + 101 = 605$ , the fourth number is  $101 + 605 = 706$ , and the fifth number is  $605 + 706 = 1311$ . As a check, we can use the rule to determine the sixth number obtaining  $706 + 1311 = 2017$ , as required.

The four missing numbers that would be written on the squares are 101, 605, 706, 1311.

