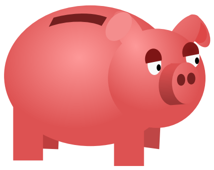

Problem of the Week

Problem C and Solution

This Problem has Value

Problem

A nickel is worth 5 cents (5¢), a dime is worth 10 cents (10¢), a quarter is worth 25 cents (25¢), and a dollar is worth 100 cents (100¢). A piggy bank contains some quarters, dimes and nickels. There are no other coins in the piggy bank. The ratio of the number of quarters to the number of dimes to the number of nickels in the piggy bank is $9:3:1$. The total value of all of the coins in the piggy bank is \$18.20. Determine the number of coins in the piggy bank.

Solution

Solution 1

Suppose the piggy bank contained one nickel. Then, using the ratio $9:3:1$, the bank would contain 9 quarters, 3 dimes and 1 nickel, 13 coins in total. The value of the 13 coins would be $25 \times 9 + 10 \times 3 + 5 \times 1 = 260$ cents or \$2.60.

Since the coins in the bank are in the ratio $9:3:1$, then we can group the coins into sets of 9 quarters, 3 dimes and 1 nickel, with each set containing 13 coins and having a value of \$2.60.

Since the total value of the coins in the bank is \$18.20 and $\frac{\$18.20}{\$2.60} = 7$, then there are 7 of these sets of coins. Since each set has 13 coins, then there are $7 \times 13 = 91$ coins in total in the piggy bank.

Solution 2

This solution uses algebra which may be beyond the solver at this point in the school year.

Suppose there are n nickels in the bank. Then, using the ratio $9:3:1$, the bank would contain $9n$ quarters, $3n$ dimes and n nickels, $13n$ coins in total.

The value of the coins would be

$$25 \times 9n + 10 \times 3n + 5 \times n = 225n + 30n + 5n = 260n \text{ cents.}$$

The total value of the coins in the bank is \$18.20 or 1820 ¢. Therefore, $260n = 1820$. Dividing both sides of the equation by 260, $n = 7$.

Since there are $13n$ coins in the bank and $n = 7$, there are $13 \times 7 = 91$ coins in the bank.

Solution 3

This solution presents the information in a table.

# of nickels	# of quarters	# of dimes	# of coins	Total value of coins
1	$9 \times 1 = 9$	$3 \times 1 = 3$	$1 + 9 + 3 = 13$	$1 \times 5 + 9 \times 25 + 3 \times 10 = 260$ ¢
2	$9 \times 2 = 18$	$3 \times 2 = 6$	$2 + 18 + 6 = 26$	$2 \times 5 + 18 \times 25 + 6 \times 10 = 520$ ¢
3	$9 \times 3 = 27$	$3 \times 3 = 9$	$3 + 27 + 9 = 39$	$3 \times 5 + 27 \times 25 + 9 \times 10 = 780$ ¢
4	$9 \times 4 = 36$	$3 \times 4 = 12$	$4 + 36 + 12 = 52$	$4 \times 5 + 36 \times 25 + 12 \times 10 = 1040$ ¢
5	$9 \times 5 = 45$	$3 \times 5 = 15$	$5 + 45 + 15 = 65$	$5 \times 5 + 45 \times 25 + 15 \times 10 = 1300$ ¢
6	$9 \times 6 = 54$	$3 \times 6 = 18$	$6 + 54 + 18 = 78$	$6 \times 5 + 54 \times 25 + 18 \times 10 = 1560$ ¢
7	$9 \times 7 = 63$	$3 \times 7 = 21$	$7 + 63 + 21 = 91$	$7 \times 5 + 63 \times 25 + 21 \times 10 = 1820$ ¢

From the table, we see that there are a total of 91 coins in the bank.

It is hoped that the solver would recognize a pattern and work with the pattern to find the solution. If the total value had been significantly greater, then this method would provide insight but would be too tedious to completely follow through.

It would also be possible to do some sort of guess and check solution to zero in on the precise solution to the problem.