



## Problem of the Week

### Problem C and Solution

### The Best Laid Plans

#### Problem

In preparing to write an examination, Stu Deus made the following observations: the exam had 20 questions, he estimated that he would spend 6 minutes per question, and the exam would take him 2 hours to complete. However, during the actual examination, Stu discovered some difficult questions which each required 15 minutes to complete. He also discovered some questions which were much easier than he expected and only took him 2 minutes per question to complete. Seven of the questions, however, still required 6 minutes each to complete. Surprisingly, Stu was still able to complete the exam in 2 hours. How many of the 20 examination questions did Stu find difficult?

#### Solution

##### Solution 1: Systematic Trial

Since 7 of the questions required 6 minutes each to complete, it took Stu  $7 \times 6 = 42$  minutes to complete these questions. The total exam took 2 hours or 120 minutes. He had  $120 - 42 = 78$  minutes to complete  $20 - 7 = 13$  questions.

Let  $d$  represent the number of difficult questions and  $e$  represent the number of easier questions. We know that  $d + e = 13$ .

Since each difficult question took 15 minutes, it took  $15d$  minutes to complete all of the difficult questions. Since each easier question took 2 minutes, it took  $2e$  minutes to complete all of the easier questions. Since Stu's total remaining time was 78 minutes,  $15d + 2e = 78$  minutes.

At this point we can pick values for  $d$  and  $e$  that add to 13 and substitute into the equation  $15d + 2e = 78$  to find the combination that works. (We can observe that  $d < 6$  since  $15 \times 6 = 90 > 78$ . If this were the case, then  $e$  would have to be a negative number.)

If  $d = 3$  then  $e = 13 - 3 = 10$ . The time to complete these would be  $15 \times 3 + 2 \times 10 = 45 + 20 = 65$  minutes and he would complete the exam in less than 2 hours.

If  $d = 4$  then  $e = 13 - 4 = 9$ . The time to complete these would be  $15 \times 4 + 2 \times 9 = 60 + 18 = 78$  minutes and he would complete the exam in exactly 2 hours.

Therefore, Stu found 4 of the questions to be more difficult and time-consuming than he expected.

Solution 2 involves algebra and equation solving.





## Solution 2: Using Algebra and Equations

Since 7 of the questions required 6 minutes each to complete, it took Stu  $7 \times 6 = 42$  minutes to complete these questions. The total exam took 2 hours or 120 minutes. He had  $120 - 42 = 78$  minutes to complete  $20 - 7 = 13$  questions.

Let  $d$  represent the number of difficult questions and  $(13 - d)$  represent the number of easier questions.

Since each difficult question took 15 minutes, it took  $15d$  minutes to complete all of the difficult questions. Since each easier question took 2 minutes, it took  $2(13 - d)$  minutes to complete all of the easier questions.

Since Stu's total remaining time was 78 minutes,

$$15d + 2(13 - d) = 78$$

$$15d + 26 - 2d = 78$$

$$13d + 26 = 78$$

Subtracting 26 from both sides:  $13d = 52$

Dividing both sides by 13:  $d = 4$

Therefore, Stu found 4 of the questions to be more difficult and time-consuming than he expected.

