Problem of the Week
Problem C and Solution
Sixteen is Just Perfect

Problem

A wall is covered with balloons. Each balloon has the number 5, 3 or 2 printed on it. You are given 7 darts to throw at the wall. If your dart breaks a balloon, then you earn the number of points printed on the balloon. If your dart does not break a balloon, then you are awarded 0 points for that shot. You win a prize if your total score is exactly 16 points on seven shots. If your total is over 16 or under 16, then you lose. Determine the number of different point combinations that can be used to win the game.

Solution

Solution 1

Let us consider cases.

1. You break three balloons with a 5 printed on them. You have a total of $3 \times 5 = 15$ points. There is no possible way to get 16 points since the other balloon values are 2 or 3. There is no way to win by breaking three (or more) balloons with a 5 printed on them.

2,. You break two balloons with a 5 printed on them. You have a total of $2 \times 5 = 10$ points. You need to get $16 - 10 = 6$ points by breaking balloons with 2 or 3 printed on them. There are two ways to do this. Break three balloons with a 2 printed on them and miss on two shots or break two balloons with 3 printed on them and miss on three shots. There are 2 ways to win if you break two balloons with a 5 printed on them.

3. You break one balloon with a 5 printed on it. You have a total of 5 points. You need to get $16 - 5 = 11$ points by breaking balloons with 2 or 3 printed on them. You cannot get 11 points breaking only balloons with a 2 printed on them and you cannot get 11 points breaking only balloons with a 3 printed on them. However, you can get 11 points by breaking one 3 and four 2's or by breaking three 3's and one 2. There are 2 ways to win if you break one balloon with a 5 printed on it.

4. You break no balloons with a 5 printed on it. You need to make 16 points by breaking only balloons with a 2 or 3 printed on them. You cannot get 16 points breaking only balloons with a 3 printed on them. You cannot get 16 points breaking only balloons with a 2 printed on them because you only have seven darts giving a maximum of 14 points. It is possible to get 16 points using combinations of 3 point and 2 point balloons. If you break two 3 point balloons and five 2 point balloons, then you win in seven shots. If you break four 3 point balloons and two 2 point balloons, then you have 16 points in six shots and would have to miss on one of your shots. There are 2 ways to win if you do not break any 5 point balloons.

There are $0 + 2 + 2 + 2 = 6$ combinations that allow you to win by getting 16 points in seven shots.

Solution 2

In this solution we will complete a chart to determine the valid possibilities.

Let the number of 5 point balloons be a, the number of 3 point balloons be b, and the number of 2 point balloons be c. We want $5a + 3b + 2c = 16$ and $a + b + c \leq 7$.

Number of 5 Point Balloons a	Number of 3 Point Balloons b	Number of 2 Point Balloons c	Total Points Scored $5a + 3b + 2c$	Number of Missed Shots Needed $7 - a - b - c$	WIN or LOSE
3	0	0	15		LOSE
3	0	1	17		LOSE
3	1	0	18		LOSE
2	2	0	16	3	WIN
2	1	1	15		LOSE
2	1	2	17		LOSE
2	0	3	16	2	WIN
1	4	0	17		LOSE
1	3	0	14		LOSE
1	3	1	16	2	WIN
1	2	2	15		LOSE
1	2	3	17		LOSE
1	1	4	16	1	WIN
0	6	0	18		LOSE
0	5	0	15		LOSE
0	5	1	17		LOSE
0	4	2	16	1	WIN
0	3	3	15		LOSE
0	3	4	17		LOSE
0	2	5	16	0	WIN
0	1	6	15		LOSE
0	0	7	14		LOSE

There are only 6 combinations that allow you to win by getting 16 points in seven shots. (In following a method like the above method, one must be careful to systematically examine all possible cases.)