



$$\sum_{i=55}^{70} i = 1000$$

Problem of the Week

Problem C and Solution

Add 'em Up

Problem

The number 1000 can be written as the sum of 16 consecutive integers? That is,

$$1000 = 55 + 56 + 57 + 58 + 59 + 60 + 61 + 62 + 63 + 64 + 65 + 66 + 67 + 68 + 69 + 70.$$

The diagram above illustrates a mathematical short form used for writing the above sum. The notation is called *Sigma Notation*. Using at least two integers, what is the minimum number of consecutive integers needed to sum to exactly 1000?

Solution

In this solution we will examine possible cases until we discover the first one that works.

1. Can 1000 be written using 2 consecutive integers?

Let n , $n + 1$ represent the two integers. Then,

$$n + (n + 1) = 1000$$

$$2n + 1 = 1000$$

$$2n = 999$$

$$n = 499.5$$

Since n is not an integer, it is not possible to write 1000 using two consecutive integers.

2. Can 1000 be written using 3 consecutive integers?

Let n , $n + 1$, $n + 2$ represent the three integers. Then,

$$n + (n + 1) + (n + 2) = 1000$$

$$3n + 3 = 1000$$

$$3n = 997$$

$$n = 332.3$$

Since n is not integer, it is not possible to write 1000 using three consecutive integers.

(Refer to the note following the solution for an alternate way to define three consecutive integers.)

3. Can 1000 be written using 4 consecutive integers?

Let n , $n + 1$, $n + 2$, $n + 3$ represent the four integers. Then,

$$n + (n + 1) + (n + 2) + (n + 3) = 1000$$

$$4n + 6 = 1000$$

$$4n = 994$$

$$n = 248.5$$

Since n is not an integer, it is not possible to write 1000 using four consecutive integers.





4. Can 1000 be written using 5 consecutive integers?

Let $n, n + 1, n + 2, n + 3, n + 4$ represent the five integers. Then,

$$\begin{aligned}n + (n + 1) + (n + 2) + (n + 3) + (n + 4) &= 1000 \\5n + 10 &= 1000 \\5n &= 990 \\n &= 198\end{aligned}$$

Since n is an integer, it is possible to write 1000 using five consecutive integers. That is, $1000 = 198 + 199 + 200 + 201 + 202$.

Since we have checked all possible numbers of consecutive integers below five and none of them worked, the minimum number of consecutive integers required to produce a sum of 1000 is five.

Note:

In the second case, when we checked to see if 1000 could be written as the sum of three consecutive integers, we could have proceeded as follows:

Let $a - 1, a, a + 1$ represent the three consecutive integers. Then,

$$\begin{aligned}(a - 1) + a + (a + 1) &= 1000 \\3a &= 1000 \\a &\doteq 333.3\end{aligned}$$

Since a is not an integer, it is not possible to write 1000 using three consecutive integers.

This “idea” is useful when we are finding the sum of an odd number of consecutive integers. If we applied the same idea to the fourth case by using $a - 2, a - 1, a, a + 1, a + 2$ to represent the five consecutive integers,

$$\begin{aligned}(a - 2) + (a - 1) + a + (a + 1) + (a + 2) &= 1000 \\5a &= 1000 \\a &= 200 \\a - 2 &= 198 \\a - 1 &= 199 \\a + 1 &= 201 \\a + 2 &= 202\end{aligned}$$

It is possible to write 1000 using five consecutive integers; namely, 198, 199, 200, 201, and 202.

For Further Thought:

What is the largest **odd** number of consecutive positive integers that can be used to sum to 1000? What is the smallest number in the list?

How would your answer change if the word positive was removed from the above sentence?

