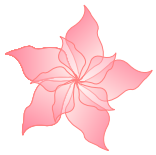
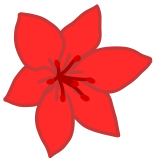


Problem of the Week
Problem C and Solution
Peculiar Perennials



Problem

A *perennial* is a plant that blooms over the spring and summer. The plant dies off over the autumn and winter but returns again the following spring. There are two known species of the POTW perennial plant, the pink ProbleminusA plant and the red ProbleminusB plant. The ProbleminusA is a pink flowering plant in its first year. The following spring it turns into a red ProbleminusB plant. That is, each ProbleminusA plant blooms as a ProbleminusB plant the next year. The ProbleminusB is a red plant. This plant blooms the following spring and also produces a pink ProbleminusA plant. That is, each ProbleminusB plant returns the following spring along with a new ProbleminusA plant. Every year this cycle reoccurs. Today in our garden we planted three ProbleminusA plants and two ProbleminusB plants. Assuming that the plants behave exactly as described, how many plants will be in the garden after 10 reproduction cycles?

Solution

Today we have 3 ProbleminusA plants and 2 ProbleminusB plants. In one year the 3 ProbleminusA plants become 3 ProbleminusB plants. In the same year the 2 ProbleminusB plants remain and produce 2 ProbleminusA plants. So after one year there are 2 ProbleminusA plants and $3 + 2 = 5$ ProbleminusB plants.

Proceeding from year 1 to year 2, the 2 ProbleminusA plants are now 2 ProbleminusB plants. The 5 ProbleminusB plants remain and produce 5 ProbleminusA plants. After two years, there are 5 ProbleminusA plants and $2 + 5 = 7$ ProbleminusB plants.

At this point we can make an observation. The number of ProbleminusA plants in a given year equals the number of ProbleminusB plants from the previous year. The number of ProbleminusB plants is the sum of the number of ProbleminusA and the number of ProbleminusB plants from the previous year.

We will use this to produce a chart.

Year Number	0	1	2	3	4	5	6	7	8	9	10
# of ProbeminusA	3	2	5	7	12	19	31	50	81	131	212
# of ProbeminusB	2	5	7	12	19	31	50	81	131	212	343

After 10 reproduction cycles, there are 212 ProbleminusA plants and 343 ProbleminusB plants for a total of 555 plants in the garden.

The number of a particular flower type in a specific year is dependent on the number of flowers of each of the types from the previous year. This is an example of a *recursion*.

In our problem,

$$\begin{aligned} & \#\text{ of ProbleminusA this year} \\ = \ & \#\text{ of ProbleminusB the previous year} \end{aligned}$$

and

$$\begin{aligned} & \#\text{ of ProbleminusB this year} \\ = \ & \#\text{ of ProbleminusA the previous year} + \#\text{ of ProbleminusB the previous year} \end{aligned}$$

A famous example of a recursion is known as the *Fibonacci Sequence*. The first two numbers in the sequence of numbers are defined. The first number, also called a term, is 1 and the second number is 1. Each remaining term in the sequence is equal to the sum of the two previous terms.

So, the third term is equal to the sum of the first and second terms, and is therefore $1 + 1 = 2$.

We can now determine the fourth term in the sequence. It will be the sum of the second and third terms, and is therefore $1 + 2 = 3$.

The fifth term in the sequence is the sum of the third and fourth terms, and is therefore $2 + 3 = 5$.

We can continue generating more terms in the sequence by applying the rule.

The first 20 Fibonacci numbers are

$$1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765$$

In our problem, the number of ProbleminusB plants is the sum of the number of ProbleminusA and ProbleminusB plants in the previous year. If we had started with only 1 ProbleminusA plant and 0 ProbleminusB plants, the number of ProbleminusB plants in years 1 to 10 would be the first 10 terms of the Fibonacci sequence.

Do an internet search to discover more about the Fibonacci sequence and recursions in general.