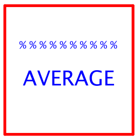

Problem of the Week
Problem C and Solution
About Average

Problem

The average of four different positive integers is 100. If the difference between the largest and smallest of these integers is as large as possible, determine the average of the other two integers.

Solution

Let S, a, b, and L represent four distinct positive integers such that $S < a < b < L$.

Since the average of the four positive integers is 100, the total sum can be determined by multiplying the average by 4. The sum of the numbers is therefore $4 \times 100 = 400$. That is, $S + a + b + L = 400$.

For the difference between the largest, L, and smallest, S, to be as large as possible, we want the smallest integer S to be as small as possible. The smallest possible positive integer is 1 so $S = 1$.

Since the sum of the four positive integers is 400 and the smallest integer, S, is 1, the sum of the remaining three integers is $a + b + L = 400 - 1 = 399$.

For L to be as large as possible, a and b must be as small as possible. The two positive integers, a and b, must be different and cannot equal 1 since the smallest of the four positive integers, S, is 1. Therefore, $a = 2$ and $b = 3$, the smallest two distinct remaining positive integers. It follows that L, the largest of the four positive integers, is $399 - 2 - 3 = 394$. (This was not required but has been provided for completeness.)

The average of the middle two positive integers, a and b, is $\frac{2+3}{2} = \frac{5}{2} = 2.5$.

For Further Thought:

How would your answer change if it was also required that the average of the four distinct positive integers was an integer greater or equal to 3?

Word Picture Solution: Ten percent above average.