



Problem of the Week

Problem C and Solution

Elf Pics



Problem

Five of Santa's elves: Alpha, Beta, Delta, Epsilon and Gamma, are lined up in alphabetical order from left to right. They can each choose one of five festive hats to wear for a photo. The hats are identical except for colour. Three of the hats are red and two of the hats are green. How many different photos can be taken?

Solution

Since the elves are already organized in alphabetical order, we are looking for the number of different ways that we can distribute the hats among the elves. We will consider cases:

1. If the first elf gets a green hat, there are four ways to give out the second green hat. Once the green hats are distributed, the remaining three elves must each get a red hat. Therefore, there are 4 ways to distribute the hats so that the first elf receives a green hat.
2. If the first elf gets a red hat and the second elf gets a green hat, there are three ways to give out the second green hat. Once the green hats are distributed, the remaining two elves must each get a red hat. Therefore, there are 3 ways to distribute the hats so that the first elf receives a red hat and the second elf receives a green hat.
3. If the first two elves each get a red hat and the third elf gets a green hat, there are two ways to give out the second green hat. Once the green hats are distributed, the remaining elf must get a red hat. Therefore, there are 2 ways to distribute the hats so that the first two elves each receive a red hat and the third elf receives a green hat.
4. If the first three elves each get a red hat and the fourth elf gets a green hat, the fifth elf must get the second green hat. Therefore, there is only 1 way to distribute the hats so that the first three elves each receive a red hat and the fourth elf receives a green hat.

There are no other cases to consider. The total number of ways to distribute the hats is the sum of the number of ways from each of the cases. Therefore, there are $4 + 3 + 2 + 1 = 10$ ways to distribute the hats. There are 10 different photos that can be taken.

