Problem of the Week

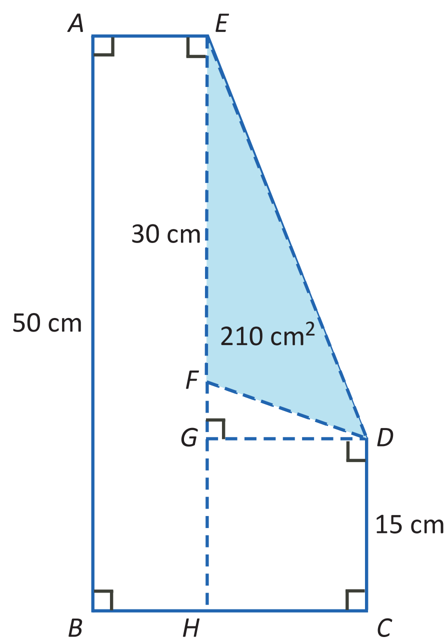
Problem C and Solution

No Longer a Rectangle

Problem

In the following slightly irregular shape, $AB = 50$ cm, $CD = 15$ cm, $EF = 30$ cm; the area of the shaded triangle, $\triangle DEF$, is 210 cm^2; and the area of the entire figure, $ABCDE$, is 1000 cm^2. Determine the length of AE.

Solution



The first task is to mark the given information on the diagram. This has been completed on the diagram to the right. EG has been extended to meet BC at H.

To find the area of a triangle, multiply the length of the base by the height and divide by 2. In $\triangle DEF$, the base, EF, has length 30 cm. The height of $\triangle DEF$ is the perpendicular distance from EF (extended) to vertex D, namely GD. The area is given. So

$$\begin{aligned} \text{Area } \triangle DEF &= \frac{30 \times GD}{2} \\ 210 &= 15 \times GD \\ 14 &= GD \end{aligned}$$

We know that $EH = AB = 50$, $GH = DC = 15$, and $EH = EF + FG + GH$. It follows that $50 = 30 + FG + 15$ and $FG = 5$ cm.

Now we can relate the total area to the areas contained inside.

$$\begin{aligned} \text{Area } ABCDE &= \text{Area } ABHE + \text{Area } CDGH + \text{Area } \triangle DFG + \text{Area } \triangle DEF \\ 1000 &= AB \times AE + GD \times DC + \frac{FG \times GD}{2} + 210 \\ 1000 &= 50 \times AE + 14 \times 15 + \frac{5 \times 14}{2} + 210 \\ 1000 &= 50 \times AE + 210 + 35 + 210 \\ 1000 &= 50 \times AE + 455 \\ 1000 - 455 &= 50 \times AE \\ 545 &= 50 \times AE \\ \frac{545}{50} &= AE \end{aligned}$$

$\therefore AE = 10.9$ cm.