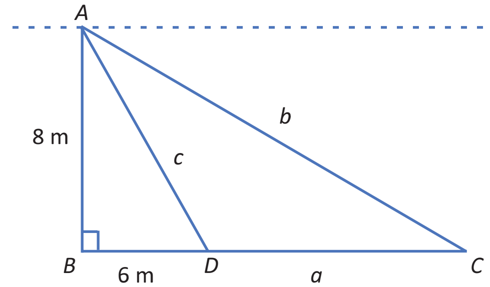
Problem of the Week
Problem C and Solution
How Far Around is It?

Problem

In the diagram, $\triangle ABC$ is a right triangle with $\angle ABC = 90^\circ$, $BD = 6$ m, $AB = 8$ m, and the area of $\triangle ADC$ is 50% more than the area of $\triangle ABD$. Determine the perimeter of $\triangle ADC$.



Solution

Let a be the length of side DC, b be the length of side AC, and c be the length of side AD. Draw a line through A parallel to BC. The distance between this line and BC is 8 m. Note that this distance is also the height of $\triangle ABD$ and $\triangle ADC$.

To find the area of a triangle, multiply the length of the base by the height and divide by 2. Therefore,

$$\text{area of } \triangle ABD = AB \times BD \div 2 = 8 \times 6 \div 2 = 24 \text{ m}^2.$$

The area of $\triangle ADC$ is 50% more than the area of $\triangle ABD$. Therefore,

$$\text{area of } \triangle ADC = (\text{area of } \triangle ABD) + \frac{1}{2}(\text{area of } \triangle ABD) = 24 + 12 = 36 \text{ m}^2.$$

But the area of $\triangle ADC = (AB)(DC) \div 2 = 8(DC) \div 2 = 4(DC)$. Therefore, $4(DC) = 36$ and $DC = 9$ m. Then $BC = BD + DC = 6 + 9 = 15$ m.

Since $\triangle ABD$ has a right angle, $AD^2 = AB^2 + BD^2 = 8^2 + 6^2 = 100$. Then $AD = \sqrt{100} = 10$, since $AD > 0$.

Also, $\triangle ABC$ has a right angle, so $AC^2 = AB^2 + BC^2 = 8^2 + 15^2 = 64 + 225 = 289$. Then $AC = \sqrt{289} = 17$, since $AC > 0$.

$$\begin{aligned} \therefore \text{The perimeter of } \triangle ADC &= a + b + c \\ &= DC + AC + AD \\ &= 9 + 17 + 10 \\ &= 36 \text{ m} \end{aligned}$$

The perimeter of $\triangle ADC$ is 36 m.